Math 240 - Assignment 3

Name _____

February 5, 2026

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 12.

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1. Find the general solution: $x\dfrac{dy}{dx} + 3y + 2x^2 = x^3 + 4x.$

2. Solve the initial value problem: $\dfrac{dy}{dx} + 4y - e^{-x} = 0, \quad y(0) = 4/3.$

3. Solve the initial value problem: $t^3\dfrac{dx}{dt} + 3t^2x = t, \quad x(2) = 0.$

4. Solve the initial value problem by treating x as the dependent variable and y as the independent variable (so that the equation is linear in x).

$$\frac{dy}{dx} = \frac{1}{x + y^2}, \quad y = 0 \text{ when } x = -2$$

5. A tank initially contains 10 gal of a salt solution in which 2 lb of salt are dissolved. A salt solution containing 1 lb of salt per gallon is pumped into the tank at 3 gal/min and the solution is uniformly mixed. The mixed solution leaves the tank at 4 gal/min. Let $A(t)$ denote the amount of salt in the tank after t minutes. Set up and solve the appropriate initial value problem to determine $A(t)$. Find the maximum amount of salt in the tank.

6. A tank initially contains 50 gal of a salt solution in which 25 lb of salt are dissolved. Starting at $t = 0$, pure water runs into the tank at 2 gal/min and the solution is uniformly mixed. The mixed solution then leaves that tank at 2 gal/min and flows into another tank initially containing 50 gal of pure water. That second tank is stirred and drained at 2 gal/min. When will the second tank contain the greatest amount of salt? (Hint: Let $A_1(t)$ be the amount of salt in tank 1 at time t. Solve the appropriate initial value problem for A_1. Then $A_1(t)/50$ will be the concentration of salt in the solution flowing into tank 2. Then set up and solve the appropriate initial value problem for tank 2. Ask if you need more help.)

7. Solve the initial value problem: $(1/x + 2y^2x)\,dx + (2yx^2 - \cos y)\,dy = 0, \quad y(1) = \pi$

8. Solve: $(ye^{xy} - 1/y)\,dx + (xe^{xy} + x/y^2)\,dy = 0$

9. Consider the equation $(y^2 + 2xy)\,dx - x^2\,dy = 0$. Show that the equation is NOT exact. Then multiply both sides of the equation by y^{-2}, and show that the new equation is exact. Solve the new equation. Is there an obvious singular solution?