MTH 240 Assignment 11 Key

1) $$f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \pi - x, & 0 < x \le \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} (\pi - x)\,dx = \frac{1}{\pi}\left(\pi x - \tfrac{1}{2}x^2\right)\Big|_0^{\pi} = \frac{1}{\pi}\left(\pi^2 - \tfrac{1}{2}\pi^2\right) = \frac{1}{2}\pi$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} (\pi - x)\cos(nx)\,dx$$

+	$\pi - x$	$\cos(nx)$
−	-1	$\frac{1}{n}\sin(nx)$
+	0	$-\frac{1}{n^2}\cos(nx)$

$$= \frac{1}{\pi}\left[\frac{\pi - x}{n}\sin(nx) - \frac{1}{n^2}\cos(nx)\right]_0^{\pi}$$

$$= -\frac{1}{n^2\pi}\left[(-1)^n - 1\right]$$

$$= \frac{1-(-1)^n}{n^2\pi}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} (\pi - x)\sin(nx)\,dx$$

+	$\pi - x$	$\sin(nx)$
−	-1	$-\frac{1}{n}\cos(nx)$
+	0	$-\frac{1}{n^2}\sin(nx)$

$$= \frac{1}{\pi}\left[\frac{x-\pi}{n}\cos(nx) - \frac{1}{n^2}\sin(nx)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\right] = \frac{1}{n}$$

$$f(x) \sim \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^2\pi}\cos nx + \frac{1}{n}\sin nx$$

2) $f(x) = x, \quad -\pi < x < \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x \, dx = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cos(nx) \, dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin(nx) \, dx = \frac{2}{\pi}\int_{0}^{\pi} x \sin(nx)\, dx$$

$+$	x	$\sin nx$
$-$	1	$-\frac{1}{n}\cos nx$
$+$	0	$-\frac{1}{n^2}\sin nx$

$$= \frac{2}{\pi}\left[-\frac{x}{n}\cos nx + \frac{1}{n^2}\sin nx\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\frac{\pi}{n}(-1)^n\right]$$

$$= -\frac{2}{n}(-1)^n$$

$$\boxed{f(x) \sim \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1}\sin(nx)}$$

3) $f(x) = 1 + \sin 5x - \cos 8x$ IS ALREADY A FOURIER SERIES:

$$a_0 = 2, \quad b_5 = 1, \quad a_8 = -1$$

ALL OTHER a_n'S AND b_n'S ARE ZERO.

4) $f(x) = x^2, \quad -2 < x < 2$

$$a_0 = \frac{1}{2}\int_{-2}^{2} x^2\,dx = \int_0^2 x^2\,dx = \frac{8}{3}$$

$$a_n = \frac{1}{2}\int_{-2}^{2} x^2 \cos\left(\frac{n\pi x}{2}\right)dx = \int_0^2 x^2 \cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \left[\frac{2x^2}{n\pi}\sin\left(\frac{n\pi x}{2}\right) + \frac{8x}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right) - \frac{16}{n^3\pi^3}\sin\left(\frac{n\pi x}{2}\right)\right]_0^2$$

+	x^2	$\cos\left(\frac{n\pi x}{2}\right)$
−	$2x$	$\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)$
+	2	$-\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right)$
−	0	$-\frac{8}{n^3\pi^3}\sin\left(\frac{n\pi x}{2}\right)$

$$= \frac{16}{n^2\pi^2}(-1)^n$$

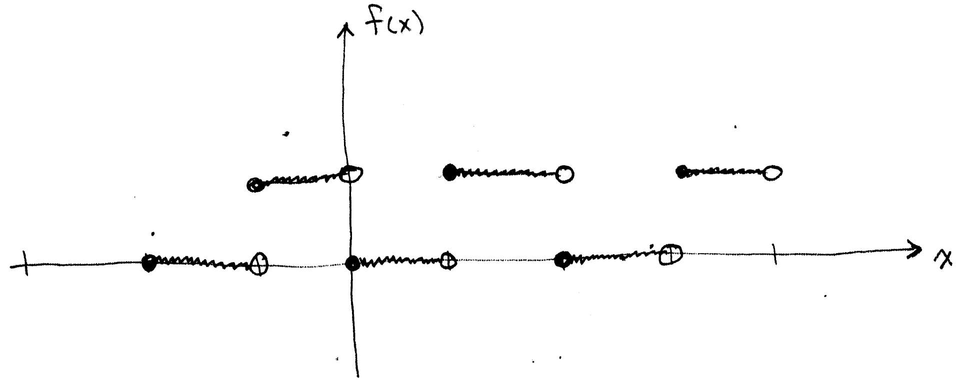
$$b_n = \frac{1}{2}\int_{-2}^{2} x^2 \sin\left(\frac{n\pi x}{2}\right)dx = 0$$

$$\boxed{f(x) \sim \frac{4}{3} + \sum_{n=1}^{\infty} \frac{16}{n^2\pi^2}(-1)^n \cos\left(\frac{n\pi x}{2}\right)}$$

5) THE FOURIER COSINE SERIES FOR $f(x) = x^2$ ON $(0,2)$ IS EXACTLY WHAT WAS COMPUTED ABOVE. SAME ANSWER.

6) $f(x) = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$

a)



b) $a_0 = \int_0^2 f(x)\, dx = \int_1^2 1\, dx = 1$

$$a_n = \int_0^2 f(x) \cos(n\pi x)\, dx = \int_1^2 \cos(n\pi x)\, dx$$

$$= \frac{1}{n\pi} \sin(n\pi x) \Big|_1^2 = 0$$

$$b_n = \int_0^2 f(x) \sin(n\pi x)\, dx = \int_1^2 \sin(n\pi x)\, dx$$

$$= -\frac{1}{n\pi} \cos(n\pi x) \Big|_1^2$$

$$= -\frac{1}{n\pi} + \frac{1}{n\pi}(-1)^n$$

$$= \frac{(-1)^n - 1}{n\pi}$$

$$\boxed{f(x) \sim \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n\pi} \sin(n\pi x)}$$