Math 240 - Quiz 4

February 16, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Solve: $x\frac{dy}{dx} + y = x^2y^2$

Bernoulli...

$\frac{dy}{dx} + \frac{1}{x}y = xy^2$

$y^{-2}\frac{dy}{dx} + \frac{1}{x}y^{-1} = x$

$u = y^{-1}$

$\frac{du}{dx} = -y^{-2}\frac{dy}{dx}$.

$-\frac{du}{dx} + \frac{1}{x}u = x$

$\frac{du}{dx} - \frac{1}{x}u = -x$

$\mu(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln|x|}$

$= \frac{1}{|x|} = \frac{1}{x}$, $x > 0$

$\frac{1}{x}u = \int \frac{1}{x}(-x)\,dx = \int -1\,dx$

$= -x + C$

$u(x) = -x^2 + Cx$

$(y(x))^{-1} = -x^2 + Cx$

$y(x) = \frac{1}{Cx - x^2}$;

$y(x) \equiv 0$ is also a solution.

2. (5 points) Solve: $x\frac{dy}{dx} = y + xe^{y/x}$, $y(1) = 1$

$\frac{dy}{dx} = \frac{y}{x} + e^{y/x}$

Homogeneous...

$u = \frac{y}{x}$, $y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$u + x\frac{du}{dx} = u + e^u$

$x\frac{du}{dx} = e^u$

$u = \frac{y}{x}$

$e^{-u}\,du = \frac{1}{x}\,dx$

$-e^{-u} = \ln|x| + C$

$e^{-u} = C - \ln|x|$

$-u = \ln(C - \ln|x|)$

$y(x) = -x\ln(C - \ln|x|)$

$y(1) = 1 \Rightarrow -1\ln(C) = 1$

$C = e^{-1}$

$y(x) = -x\ln(e^{-1} - \ln|x|)$