MTH 240 Assignment #9

1.)

a) $3x'' + x' + 5x = 5\cos 4t; \quad x(0) = -2, \quad x'(0) = 1$

b)
c) } See attached sheet.
d)

e) $k = 5,\ m = 3,\ \gamma = 4,\ b = 1 \Rightarrow$ Gain factor $= \dfrac{1}{\sqrt{1865}}$

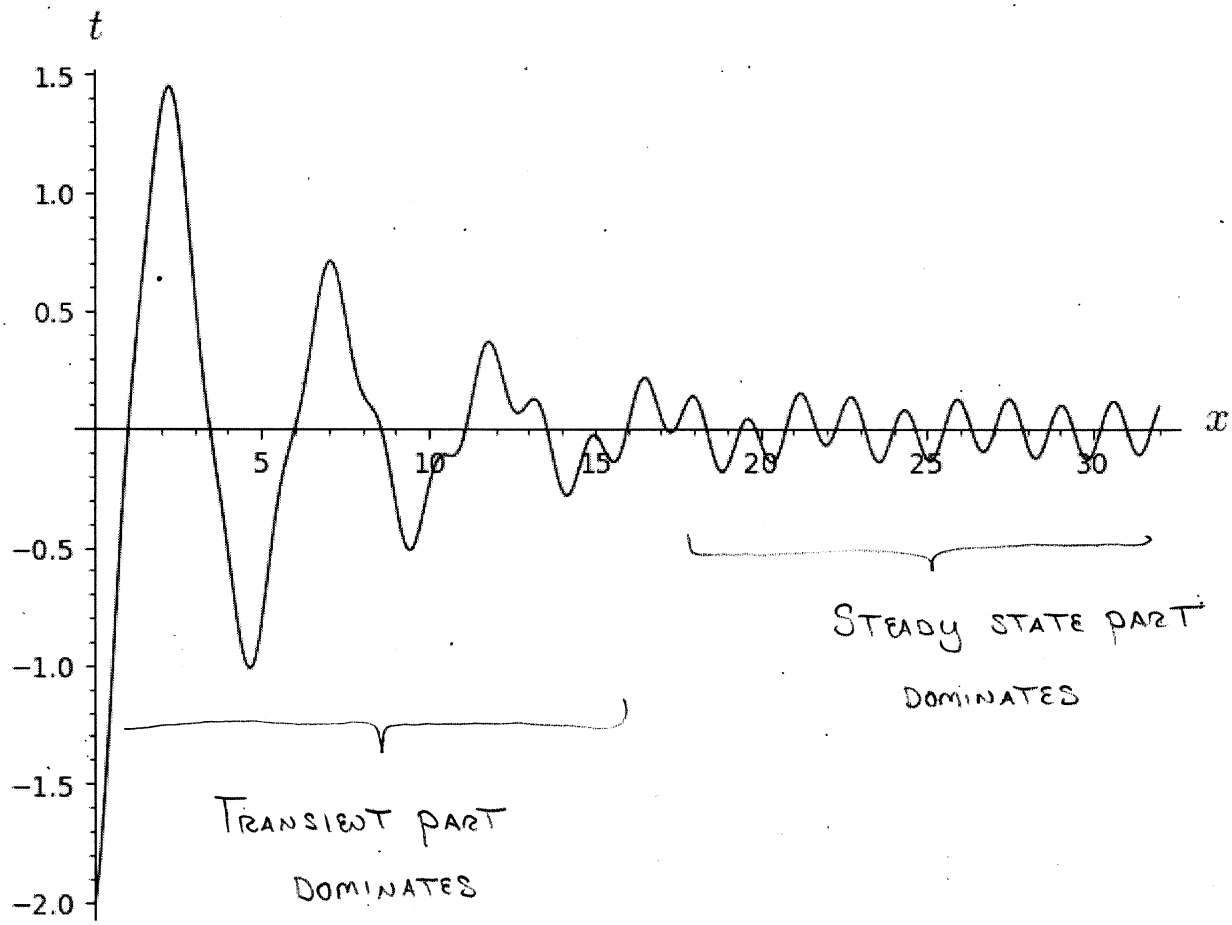
≈ 0.023156

f) $\boxed{\gamma_r = \sqrt{\frac{5}{3} - \frac{1}{18}} \approx 1.269296}$

```
t=var("t")
x=function("x")(t)
de=3*diff(x,t,2)+diff(x,t)+5*x==5*cos(4*t)
desolve(de,x,[0,-2,1])
```

Solution is...

```
1439/22007*sqrt(59)*e^(-1/6*t)*sin(1/6*sqrt(59)*t) -
703/373*cos(1/6*sqrt(59)*t)*e^(-1/6*t) - 43/373*cos(4*t) + 4/373*sin(4*t)
```



2.)

a) $(x-2)y'' - xy' + 9(x+2)y = 0$

$$y'' - \frac{x}{x-2}y' + \frac{9(x+2)}{x-2}y = 0$$

$x = 2$ IS THE ONLY SING. PT.

DISTANCE FROM $x = 2$ TO $x = 0$ IS 2 UNITS.

$$\boxed{R \geq 2}$$

b) $(x^2 - x)y'' - y' + 5(x+3)y = 0$

$$y'' - \frac{1}{x(x-1)}y' + \frac{5(x+3)}{x(x-1)}y = 0$$

$x = 0$ AND $x = 1$ ARE SING PTS.

DISTANCE FROM $x = 0$ TO $x = 0$ IS 0 UNITS.

$$\boxed{R \geq 0}$$

c) $(x^2+8)y'' + 2x^2y' + 6y = 0$

$$y'' + \frac{2x^2}{x^2+8}y' + \frac{6}{x^2+8}y = 0$$

$x^2 + 8 = 0 \Rightarrow x = \pm\sqrt{8}\,i$. $\quad x = \pm\sqrt{8}\,i$ ARE SING. PTS.

DISTANCE FROM $x = \pm\sqrt{8}\,i$ TO $x = 0$ IS $\sqrt{8}$ UNITS.

$$\boxed{R \geq \sqrt{8}}$$

3.) $y'' - (1+x)y = 0$

THERE ARE NO SING. PTS. $\boxed{R = \infty}$

$$y = \sum_{n=0}^{\infty} a_n x^n, \quad y' = \sum_{n=1}^{\infty} n a_n x^{n-1}, \quad y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$0 = y'' - y - xy$$

$$= \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=0}^{\infty} a_n x^n - \sum_{n=0}^{\infty} a_n x^{n+1}$$

$n \leftrightarrow n+2$ $\qquad\qquad$ $n \leftrightarrow n-1$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} a_n x^n - \sum_{n=1}^{\infty} a_{n-1} x^n$$

$$= 2a_2 - a_0 + \sum_{n=1}^{\infty} \left[(n+2)(n+1) a_{n+2} - a_n - a_{n-1} \right] x^n$$

$2a_2 - a_0 = 0$ AND $(n+2)(n+1) a_{n+2} - a_n - a_{n-1} = 0; \quad n = 1, 2, 3, \ldots$

a_0 ARBITRARY

a_1 ARBITRARY

$a_2 = \frac{1}{2} a_0$

$$a_{n+2} = \frac{a_n + a_{n-1}}{(n+2)(n+1)}; \quad n = 1, 2, 3, \ldots$$

4.) $(1-x^2)y'' - 6xy' - 4y = 0$

SING. PTS. ARE $x = \pm 1$. THE DISTANCE FROM $x = \pm 1$ TO $x = 0$ IS 1 UNIT, SO $\boxed{R \geq 1}$.

$$y = \sum_{n=0}^{\infty} a_n x^n, \quad y' = \sum_{n=1}^{\infty} n a_n x^{n-1}, \quad y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$0 = y'' - x^2 y'' - 6xy' - 4y$$

$$= \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1) a_n x^n - \sum_{n=1}^{\infty} 6n a_n x^n - \sum_{n=0}^{\infty} 4 a_n x^n$$

$n \leftrightarrow n+2$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} n(n-1) a_n x^n - \sum_{n=0}^{\infty} 6n a_n x^n - \sum_{n=0}^{\infty} 4 a_n x^n$$

$$= \sum_{n=0}^{\infty} \left[(n+2)(n+1) a_{n+2} - n(n-1) a_n - 6n a_n - 4a_n \right] x^n$$

$$(n+2)(n+1) a_{n+2} - n(n-1) a_n - 6n a_n - 4 a_n = 0; \quad n = 0, 1, 2, 3, \ldots$$

$$a_{n+2} = \frac{n^2 + 5n + 4}{(n+2)(n+1)} a_n; \quad n = 0, 1, 2, \ldots$$

$$\boxed{a_{n+2} = \frac{n+4}{n+2} a_n; \quad n = 0, 1, 2, 3, \ldots}$$

a_0 ARBITRARY

a_1 ARBITRARY

CONTINUED →

a_0 ARBITRARY

$a_2 = 2a_0$

$a_4 = \frac{3}{2}a_2 = 3a_0$

$a_6 = \frac{4}{3}a_4 = 4a_0$

$\vdots$

a_1 ARBITRARY

$a_3 = \frac{5}{3}a_1$

$a_5 = \frac{7}{5}a_3 = \frac{7}{3}a_1$

$a_7 = \frac{9}{7}a_5 = \frac{9}{3}a_1$

$\vdots$

$$\boxed{\begin{aligned} y_1(x) &= a_0\left(1 + 2x^2 + 3x^4 + 4x^6 + \cdots\right) \\ y_2(x) &= a_1\left(x + \frac{5}{3}x^3 + \frac{7}{3}x^5 + \frac{9}{3}x^7 + \cdots\right) \end{aligned}}$$