# MTH 240 - Assignment #10

1) $y'' - 3y' + 2y = \cos t; \quad y(0) = 0, \; y'(0) = -1$

$$s^2 Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{-1} - 3sY(s) + 3\cancel{y(0)}^{0} + 2Y(s) = \frac{s}{s^2+1}$$

$$\underbrace{(s^2 - 3s + 2)}_{(s-2)(s-1)} Y(s) = \frac{s}{s^2+1} - 1$$

$$Y(s) = \frac{s}{(s^2+1)(s-2)(s-1)} - \frac{1}{(s-2)(s-1)}$$

} PFD by Sage

$$= \frac{1}{10}\frac{s-3}{s^2+1} + \frac{1}{2}\frac{1}{s-1} - \frac{3}{5}\frac{1}{s-2}$$

$$\boxed{y(t) = \frac{1}{10}\cos t - \frac{3}{10}\sin t + \frac{1}{2}e^t - \frac{3}{5}e^{2t}}$$

2.) $y'' - 4y = 4t - 8e^{-2t}$ ; $y(0) = 0$, $y'(0) = 5$

$$s^2 Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{5} - 4Y(s) = \frac{4}{s^2} - \frac{8}{s+2}$$

$$(s^2-4)Y(s) - 5 = \frac{4}{s^2} - \frac{8}{s+2}$$

$$Y(s) = \frac{4}{s^2(s+2)(s-2)} - \frac{8}{(s+2)^2(s-2)} + \frac{5}{(s+2)(s-2)}$$

$$= \frac{-1}{s+2} + \frac{1}{s-2} + \frac{2}{(s+2)^2} - \frac{1}{s^2}$$

PFD by Sage

$$y(t) = -e^{-2t} + e^{2t} + 2te^{-2t} - t$$

3.) $y'' - 6y' + 9y = t^2 e^{3t}$ ; $y(0) = 2$, $y'(0) = 6$

$$s^2 Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(0) + 9Y(s) = \frac{2}{(s-3)^3}$$

$$\underbrace{(s^2 - 6s + 9)}_{(s-3)^2} Y(s) - 2s + 6 = \frac{2}{(s-3)^3}$$

$$Y(s) = \frac{2}{(s-3)^5} + \frac{2s-6}{(s-3)^2}$$

$$Y(s) = \frac{2}{(s-3)^5} + \frac{2}{(s-3)} = \frac{2}{4!}\frac{4!}{(s-3)^5} + 2\cdot\frac{1}{(s-3)}$$

$$y(t) = \frac{1}{12} t^4 e^{3t} + 2e^{3t}$$

4.) $y'' + 4y' + 4y = t^3 e^{-2t}$; $y(0) = 5$, $y'(0) = -10$

$$s^2 Y(s) - s\cancel{y(0)}^{5} - \cancel{y'(0)}^{-10} + 4sY(s) - 4\cancel{y(0)}^{5} + 4Y(s) = \frac{3!}{(s+2)^4}$$

$$\underbrace{(s^2 + 4s + 4)}_{(s+2)^2} Y(s) - 5s - 10 = \frac{6}{(s+2)^4}$$

$$Y(s) = \frac{6}{(s+2)^6} + \frac{5(s+2)}{(s+2)^2} = \frac{6}{5!}\,\frac{5!}{(s+2)^6} + 5 \cdot \frac{1}{s+2}$$

$$\boxed{y(t) = \frac{1}{20} t^5 e^{-2t} + 5e^{-2t}}$$

5.) $x' = -x + y, \quad x(0) = 0$

$y' = 2x, \quad y(0) = 1$

$s X(s) = -X(s) + Y(s) \longrightarrow (s+1) X(s) = Y(s)$

$s Y(s) - 1 = 2 X(s)$

$s(s+1) X(s) - 2 X(s) = 1$

$(s^2 + s - 2) X(s) = 1$

$$X(s) = \frac{1}{(s+2)(s-1)}$$

$$Y(s) = \frac{s+1}{(s+2)(s-1)}$$

$$X(s) = \frac{1}{(s+2)(s-1)} = \frac{-1/3}{s+2} + \frac{1/3}{s-1}$$

$$Y(s) = \frac{s+1}{(s+2)(s-1)} = \frac{1/3}{s+2} + \frac{2/3}{s-1}$$

PFD by Cover up

$$x(t) = -\frac{1}{3}e^{-2t} + \frac{1}{3}e^{t}$$

$$y(t) = \frac{1}{3}e^{-2t} + \frac{2}{3}e^{t}$$

6.) $x''' + x'' - 6x' = e^{4t}$; $x(0)=0$, $x'(0)=1$, $x''(0)=1$

$$s^3X(s) - s^2\cancel{x(0)}^{0} - s\cancel{x'(0)}^{1} - \cancel{x''(0)}^{1} + s^2X(s) - s\cancel{x(0)}^{0} - \cancel{x'(0)}^{1} - 6sX(s) + 6\cancel{x(0)}^{0} = \frac{1}{s-4}$$

$$\underbrace{(s^3+s^2-6s)}_{s(s+3)(s-2)}X(s) - s - 2 = \frac{1}{s-4}$$

$$X(s) = \frac{1}{(s-4)s(s+3)(s-2)} + \frac{s+2}{s(s+3)(s-2)}$$

$$= \frac{s^2-2s-7}{s(s-4)(s+3)(s-2)}$$

PFD by Cover up

$$= \frac{-7/24}{s} + \frac{1/56}{s-4} - \frac{8/105}{s+3} + \frac{7/20}{s-2}$$

$$x(t) = -\frac{7}{24} + \frac{1}{56}e^{4t} - \frac{8}{105}e^{-3t} + \frac{7}{20}e^{2t}$$