

Math 240 - Assignment 4

September 19, 2024

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due September 26.

1. Solve: $\frac{dy}{dx} = y + e^{2x}y^3$.

2. Solve: $\frac{dy}{dx} = \frac{x^2 - y^2}{3xy}$.

3. Solve: $yy'' + (y')^2 = 0$.

4. Solve: $xy'' = 1 + y; \quad x, y > 0$.

5. Consider the following ODE: $4x^2y'' - 3y = 0$.

(a) Verify that $y_1(x) = x^{3/2}$ and $y_2(x) = x^{-1/2}$ are solutions for $x > 0$.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(0, \infty)$.

(c) Find the unique solution that satisfies $y(4) = 24$ and $y'(4) = 5$.

6. Consider the following ODE: $yy'' + (y')^2 = 0$.

(a) Verify that $y_1(x) = 1$ and $y_2(x) = \sqrt{x}$ are solutions.

(b) Show that $y_1(x) + y_2(x)$ is not a solution.

(c) Why should you not expect the sum of solutions to be a solution?

7. Find the general solution: $y'' - 4y' - 21y = 0$.

8. Find the general solution: $2y'' + 6y' + 2y = 0$.

9. Find a differential equation that has the general solution given below.

$$y(x) = c_1e^{-5x} + c_2e^{3x} + c_3xe^{3x}$$

10. Solve the initial value problem: $y'' - 4y' + 4y = 0; \quad y(1) = 1, y'(1) = 1$.