Math 240 - Test 3A

November 19, 2020

Name ______________________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Use Laplace transform techniques to solve.

$$y'' + 4y' + 4y = t^3 e^{-2t}; \quad y(0) = 5,\, y'(0) = -10$$

2. (10 points) State the recurrence relation that describes the coefficients of the power series solution, and state the guaranteed (by our theorem) radius of convergence.

$$(x^2 - 2)y'' + xy' - 16y = 0$$

Math 240 - Test 3B

November 19, 2020

Name ____________________

Score ________

Show all work to receive full credit. Supply explanations where necessary. This test is due December 1 by email. **You must work individually on this test.**

1. (15 points) Find a power series solution. Then refer to the Common Infinite Series handout to find the interval of convergence and a familiar expression for your solution.

$$(x-10)y' + y = 0$$

2. (20 points) Consider the following first-order linear equation.

$$x^3 y' = 2y$$

(a) Find a power series solution.

(b) Use the techniques of section 1.5 to find the general solution.

(c) Compare your solutions. Why did the power series approach fail to provide the general solution?

3. (8 points) Use the definition of the Laplace transform to find the transform of f.

$$f(t) = \begin{cases} 1 - t, & 0 \leq t \leq 1 \\ 0, & t > 1 \end{cases}$$

4. (15 points) Use Laplace transform techniques to solve the following initial value problem. You may use technology for any partial fraction decompositions, but do everything else by hand.

$$y'' + 6y' + 18y = \cos 2t; \quad y(0) = 1,\ y'(0) = -1$$

5. (8 points) Use Laplace transform techniques to solve the following system of initial value problems.

$$x' = x + 2y, \quad x(0) = 0$$
$$y' = x + e^{-t}, \quad y(0) = 0$$

6. (14 points) Let $F(s) = \dfrac{3}{s(s+5)}$.

(a) Find the inverse Laplace transform by first computing the partial fraction decomposition (by hand).

(b) Find the inverse Laplace transform by using Theorem 2 on page 284.

(c) Apply the convolution theorem to find the inverse Laplace transform.