Math 236 - Test 2

March 11, 2026

Name key

Score

Show all work to receive full credit. Supply explanations when necessary. You may use your calculator to obtain any RREF.

1. (10 points) Consider the following set of matrices in $\mathcal{M}_{2\times 2}$.

$$S = \left\{ A_1 = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} 4 & -2 \\ 2 & -2 \end{pmatrix}, A_3 = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, A_4 = \begin{pmatrix} 5 & 0 \\ 1 & -1 \end{pmatrix} \right\}$$

(a) Show that the set S is linearly dependent set.

$$c_1 A_1 + c_2 A_2 + c_3 A_3 + c_4 A_4 = 0$$

$$\Downarrow$$

$$\begin{aligned} c_1 + 4c_2 + c_3 + 5c_4 &= 0 \\ 2c_1 - 2c_2 + 2c_3 &= 0 \\ c_1 + 2c_2 + 3c_3 + c_4 &= 0 \\ c_1 - 2c_2 + c_3 - c_4 &= 0 \end{aligned} \xrightarrow{\text{RREF}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

System has inf. many solutions. In fact, $\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 1 \\ 1 \end{pmatrix} c_4$ for any $c_4 \in \mathbb{R}$.

(b) Write any one of the matrices in S as a linear combination of the others.

Choose $c_4 = 1$. Then we have $-2A_1 - A_2 + A_3 + A_4 = 0$.

$$\boxed{A_2 = -2A_1 + A_3 + A_4}$$

2. (8 points) Find a basis for the row space, a basis for the column space, and the rank of the matrix A.

$$A = \begin{pmatrix} 1 & 4 & 5 & 1 & 0 \\ -2 & 0 & 6 & -1 & -6 \\ 3 & 1 & -7 & 3 & 11 \end{pmatrix} \xrightarrow{\text{RREF}} \begin{pmatrix} 1 & 0 & -3 & 0 & 2 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\boxed{\text{Rank}(A) = 3}$$

Basis for row space $= B_R = \langle (1,0,-3,0,2), (0,1,2,0,-1), (0,0,0,1,2) \rangle$

Basis for column space $= B_C = \left\langle \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \right\rangle$

3. (8 points) Let W be the subspace of $\mathcal{P}_3$ defined by

$$W = \{ax^3 + bx^2 + cx + d : \underbrace{2a + b + c = 0}_{c=-2a-b} \text{ and } \underbrace{2b + d = 0}_{d=-2b}\}.$$

(a) Find a basis for W.

$$W = \{ax^3 + bx^2 + (-2a-b)x + (-2b) : a, b \in \mathbb{R}\}$$

$$= \{a(x^3 - 2x) + b(x^2 - x - 2) : a, b \in \mathbb{R}\}$$

$$= \text{span}(\underbrace{\{x^3 - 2x, x^2 - x - 2\}}) \qquad \boxed{B = \langle x^3 - 2x, x^2 - x - 2 \rangle}$$

INDEP. SINCE ONE IS NOT A MULTIPLE OF THE OTHER.

(b) What is the dimension of W?

$$\dim(W) = \boxed{2}$$

(c) Represent $p(x) = -2x^3 - 6x^2 + 10x + 12$ in terms of your basis.

$$-2(x^3 - 2x) - 6(x^2 - x - 2) = -2x^3 - 6x^2 + 10x + 12$$

$$\boxed{\text{Rep}_B(p(x)) = \begin{pmatrix} -2 \\ -6 \end{pmatrix}_B}$$

4. (4 points) A matrix has 5 rows and 9 columns. Which set must be dependent, its set of rows or its set of columns? Carefully explain your reasoning.

THE RANK CANNOT POSSIBLY EXCEED 5 BECAUSE THERE ARE ONLY 5 ROWS, THEREFORE, THERE CANNOT BE MORE THAN 5 INDEPENDENT COLUMNS. THE COLUMNS MUST BE A DEPENDENT SET.

5. (8 points) Let $W = \text{span}(\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\})$, where

$$\vec{v}_1 = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 4 \\ -9 \\ -1 \end{pmatrix}, \quad \vec{v}_4 = \begin{pmatrix} -2 \\ 7 \\ 3 \end{pmatrix}.$$

(a) Briefly explain why the vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, and $\vec{v}_4$ cannot be linearly independent.

$\dim(\mathbb{R}^3) = 3 \Rightarrow$ THE DIMENSION OF W CANNOT EXCEED 3.

$W \subseteq \mathbb{R}^3$

(b) Find a basis for span(W).

MAKE THE VECTORS BE COLUMNS OF A MATRIX AND USE RREF...

$$\begin{pmatrix} 2 & -1 & 4 & -2 \\ -5 & 3 & -9 & 7 \\ 1 & 1 & -1 & 3 \end{pmatrix} \xrightarrow{\text{RREF}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 10/3 \\ 0 & 0 & 1 & 1/3 \end{pmatrix}}$$

$\vec{v}_1, \vec{v}_2, \vec{v}_3$ ARE LINEARLY INDEP.

AND $\vec{v}_4 = \frac{10}{3}\vec{v}_2 + \frac{1}{3}\vec{v}_3$.

$\boxed{\text{BASIS FOR } W = \langle \vec{v}_1, \vec{v}_2, \vec{v}_3 \rangle}$

6. (8 points) Suppose $h : \mathcal{P}_2 \to \mathcal{M}_{2\times 2}$ is a homomorphism with

$$h(x^2+2x+1) = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}, \quad h(x^2-3) = \begin{pmatrix} -1 & 5 \\ 0 & -3 \end{pmatrix}, \quad h(x+1) = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}.$$

Determine $h(5x^2 - x - 12)$.

$5x^2 - x - 12 = a(x^2+2x+1) + b(x^2-3) + c(x+1)$

$\Rightarrow \quad a+b=5, \quad 2a+c=-1, \quad a-3b+c=-12$

$b = 5-a. \qquad c = -1-2a \qquad a - 3(5-a) + (-1-2a) = -12$

$2a = 4$

$b = 3 \qquad c = -5 \qquad a = 2$

$$h(5x^2-x-12) = 2\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} + 3\begin{pmatrix} -1 & 5 \\ 0 & -3 \end{pmatrix} - 5\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} -2 & 12 \\ -3 & -15 \end{pmatrix}}$$

7. (10 points) Suppose $\vec{x}_1, \vec{x}_2, \ldots \vec{x}_k$ are a linearly independent vectors in the vector space V. Also suppose that $f : V \to W$ is an isomorphism. Prove that $f(\vec{x}_1), f(\vec{x}_2), \ldots f(\vec{x}_k)$ are linearly independent in W.

ASSUME $\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_k$ ARE LIN. INDEP. AND SUPPOSE

$$c_1 f(\vec{x}_1) + c_2 f(\vec{x}_2) + \cdots + c_k f(\vec{x}_k) = \vec{0}.$$

SINCE f IS AN ISOMORPHISM, THE LEFT HAND SIDE IS

$f(c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_k\vec{x}_k)$. SO, $f(\vec{c}_1\vec{x}_1 + \cdots + c_k\vec{x}_k) = \vec{0}$,

AND $\vec{0} = f(\vec{0})$ BECAUSE ISOMORPHISMS MAP $\vec{0}$ TO $\vec{0}$.

NOW WE HAVE $f(c_1\vec{x}_1 + \cdots + c_k\vec{x}_k) = f(\vec{0})$ AND SINCE f IS 1-1,

WE MUST HAVE $c_1\vec{x}_1 + \cdots + c_k\vec{x}_k = \vec{0}$. FINALLY, SINCE

$\vec{x}_1, \ldots, \vec{x}_k$ ARE INDEP., IT FOLLOWS THAT $c_1 = c_2 = \cdots = c_k = 0$.

Follow up: If f is a homomorphism, but not an isomorphism, the result is no longer true. What goes wrong in your proof?

THE PART WHERE WE USE 1-1 WOULD NOT BE VALID.

$f(c_1\vec{x}_1 + \cdots + c_k\vec{x}_k) = \vec{0}$ WOULD ONLY IMPLY THAT $c_1\vec{x} + \cdots + c_k\vec{x}_k$ IS IN THE NULL SPACE OF f.

8. (6 points) Suppose $g : U \to V$ and $f : V \to W$ are isomorphisms. Show that the composition of functions, $f \circ g : U \to W$, is one-to-one and onto. (Recall that $(f \circ g)(x)$ means $f(g(x))$.)

1-1: SUPPOSE $f(g(\vec{x})) = f(g(\vec{y}))$.

SINCE f IS 1-1, WE MUST HAVE $g(\vec{x}) = g(\vec{y})$.

SINCE g IS 1-1, WE MUST HAVE $\vec{x} = \vec{y}$. ✓

ONTO: LET $\vec{\omega}$ BE ANY ARBITRARY ELEMENT OF W.

SINCE f IS ONTO, $\exists\ \vec{v} \in V$ s.t. $f(\vec{v}) = \vec{\omega}$.

SINCE g IS ONTO V, $\exists\ \vec{u} \in U$ s.t. $g(\vec{u}) = \vec{v}$

WITH THIS $\vec{u}$, WE HAVE $f(g(\vec{u})) = \vec{\omega}$. ✓

9. (10 points) Define $f : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ 2x \end{pmatrix}.$$

Show that f is an isomorphism.

ONTO: Suppose $\begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{R}^2$. WE WANT $x+y=a$ & $2x=b$.

$x = \frac{b}{2}$, $y = a - \frac{b}{2}$

WITH THIS CHOICE, WE HAVE

$$f\begin{pmatrix} x \\ y \end{pmatrix} = f\begin{pmatrix} b/2 \\ a-b/2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}. \checkmark$$

ONE-TO-ONE:

Suppose $f\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = f\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$.

THEN $\begin{pmatrix} x_1+y_1 \\ 2x_1 \end{pmatrix} = \begin{pmatrix} x_2+y_2 \\ 2x_2 \end{pmatrix}$.

$2x_2 = 2x_1 \Rightarrow x_1 = x_2$

$x_1 + y_1 = x_2 + y_2$

$\Rightarrow x_1 + y_1 = x_1 + y_2$

$\Rightarrow y_1 = y_2$.

$\therefore$ WE HAVE $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$. $\checkmark$

LINEAR:

$$f\left(\alpha\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \beta\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) = f\begin{pmatrix} \alpha x_1 + \beta x_2 \\ \alpha y_1 + \beta y_2 \end{pmatrix} = \begin{pmatrix} (\alpha x_1 + \beta x_2) + (\alpha y_1 + \beta y_2) \\ 2(\alpha x_1 + \beta x_2) \end{pmatrix}$$

$$= \cdots = \begin{pmatrix} \alpha x_1 + \alpha y_1 + \beta x_2 + \beta y_2 \\ \alpha(2x_1) + \beta(2x_2) \end{pmatrix} = \alpha\begin{pmatrix} x_1+y_1 \\ 2x_1 \end{pmatrix} + \beta\begin{pmatrix} x_2+y_2 \\ 2x_2 \end{pmatrix}$$

$$= \alpha f\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \beta f\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}. \checkmark$$

10. (8 points) Define the homomorphism $h : \mathcal{P}_2 \to \mathbb{R}^3$ by

$$h(ax^2 + bx + c) = \begin{pmatrix} a - c \\ 0 \\ b + 2c \end{pmatrix}.$$

(a) Before doing any work on this problem, determine the sum of the rank and nullity of h. Very briefly say how you know.

RANK + NULLITY = DIMENSION OF DOMAIN = $\dim(\mathcal{P}_2)$

$= \boxed{3}$

(b) Determine a basis for the range space of h.

$$\mathcal{R}(h) = \left\{ \begin{pmatrix} a-c \\ 0 \\ b+2c \end{pmatrix} : a,b,c \in \mathbb{R} \right\}$$

LINEAR COMBO OF 1ST TWO!

$$= \left\{ a\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} : a,b,c \in \mathbb{R} \right\}$$

$$= \text{span}\left(\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \right)$$

$$\boxed{B_R = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle}$$

THESE BASIS VECTORS ARE CLEARLY LIN. INDEP. SINCE ONE IS NOT A MULTIPLE OF THE OTHER.

(c) Determine a basis for the null space of h.

$a - c = 0, \quad b + 2c = 0$

$a = c, \quad b = -2c$

$$\mathcal{N}(h) = \{ cx^2 - 2cx + c : c \in \mathbb{R} \}$$

$$= \text{span}(\{ x^2 - 2x + 1 \})$$

$$\boxed{B_{\mathcal{N}} = \langle x^2 - 2x + 1 \rangle}$$

The following problems are due March 23. You must work on your own. If you are not familiar with matrix multiplication, acquaint yourself with it before continuing. If necessary, a simple Google search should provide sufficient details.

11. (2 points) Just for a quick warm-up, compute the product AB by hand. (After you finish your computation, you may check your result with a computer or calculator.)

$$A = \begin{pmatrix} 2 & 4 & -5 \\ -3 & 4 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 7 \\ 2 & 3 \\ -4 & 5 \end{pmatrix}$$

$$AB = \begin{pmatrix} -2+8+20 & 14+12-25 \\ 3+8+8 & -21+12-10 \end{pmatrix} = \boxed{\begin{pmatrix} 26 & 1 \\ 19 & -19 \end{pmatrix}}$$

12. (2 points) Suppose $A \in \mathcal{M}_{m\times n}$ and $b \in \mathbb{R}^n$. The matrix-vector product Ab can be computed by using usual matrix multiplication, treating the vector b as an $n \times 1$ matrix.

(a) Compute Ab when $A = \begin{pmatrix} 2 & 1 & 3 \\ 9 & 0 & -2 \\ 2 & 5 & -3 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$.

$$Ab = \begin{pmatrix} 6-1+12 \\ 27+0-8 \\ 6-5-12 \end{pmatrix} = \boxed{\begin{pmatrix} 17 \\ 19 \\ -11 \end{pmatrix}}$$

(b) It is often convenient to think of the product Ab as a linear combination of the columns of A with the coefficients from b. Use this approach to recompute Ab from above.

$$Ab = \boxed{3\begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix} + (-1)\begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} + 4\begin{pmatrix} 3 \\ -2 \\ -3 \end{pmatrix}} = \begin{pmatrix} 6-1+12 \\ 27+0-8 \\ 6-5-12 \end{pmatrix} = \boxed{\begin{pmatrix} 17 \\ 19 \\ -11 \end{pmatrix}}$$

13. (4 points) Consider the matrix $A = \begin{pmatrix} 1 & 0 & 3 & 1 & -2 \\ 2 & 3 & -9 & 1 & 0 \\ -1 & 5 & -28 & 2 & -10 \\ 3 & -2 & 19 & 4 & -10 \end{pmatrix}$.

(a) Compute the RREF of A (use technology!). If applicable, remove any rows that consist entirely of zeros, and call the remaining matrix R.

$$R = \begin{pmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & -5 & 0 & 0 \\ 0 & 0 & 0 & 1 & -4 \end{pmatrix}$$

(b) Use your RREF to determine the <u>linearly independent columns of the original</u> matrix A. Put those columns into a matrix and call it C.

1ST, 2ND, 4TH

$$C = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ -1 & 5 & 2 \\ 3 & -2 & 4 \end{pmatrix}$$

(c) The matrix product CR must be defined. Explain how you know.

NUMBER OF COLUMNS OF C = NUMBER OF ROWS OF R = RANK OF A.

\# OF COLUMNS OF C = COLUMN RANK OF A

\# OF ROWS OF R = ROW RANK OF A

COLUMN RANK = ROW RANK.

(d) Compute CR and compare it to A.

$CR = A$

14. (12 points) Each of the elementary row operations that we use in Gauss-Jordan elimination correspond to matrix multiplication. The matrices that "perform" the elementary row operations are called *elementary matrices*, and there are three general types, one for each row operation. For example, if $A \in \mathcal{M}_{3\times n}$ and

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

then the product EA is the matrix A with its 2nd and 3rd rows swapped. Similarly, if

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

then the product EA is the matrix A with it 3rd row replaced by -2 times the original 3rd row. Finally, when multiplied on the left of A, the matrix

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}$$

would leave rows 1 and 2 unchanged, but would replace the 3rd row of A with 5 times the 1st row plus the 3rd row.

Experiment with these types of matrix multiplications! Continue with this problem after you feel comfortable. Feel free to use a computer or calculator to check all of your multiplications.

Continued ⟶

(a) Let $A = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 2 & 0 \\ 2 & 1 & 1 \end{pmatrix}$.

By hand, convert A to RREF, and keep track (in order) of the sequence of elementary matrices that correspond to your row operations. When all is said and done, you should have elementary matrices $E_1, E_2, \ldots E_k$ with the property that

$$E_k \cdots E_2 E_1 A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

① $R_2 = -4R_1 + R_2$
② $R_3 = -2R_1 + R_3$

$$\xrightarrow{} \begin{pmatrix} 1 & 3 & 2 \\ 0 & -10 & -8 \\ 0 & -5 & -3 \end{pmatrix} \xrightarrow{③\ R_2 = -\frac{1}{10} R_2} \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 0.8 \\ 0 & -5 & -3 \end{pmatrix}$$

④ $\xrightarrow{R_3 = 5R_2 + R_3} \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 0.8 \\ 0 & 0 & 1 \end{pmatrix}$ ⑤ $R_2 = -0.8R_3 + R_2$ ⑥ $R_1 = -2R_3 + R_1$ $\longrightarrow \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

⑦ $\xrightarrow{R_1 = -3R_2 + R_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

7 ROW OPS ⇒ 7 ELEM. MATRICES

IN ORDER OF ROW OPS, THE ELEMENTARY MATRICES ARE:

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix},\quad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/10 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{pmatrix},\quad E_5 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -4/5 \\ 0 & 0 & 1 \end{pmatrix},\quad E_6 = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_7 = \begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

FYI, THE PRODUCT $E_7 E_6 \cdots E_1$ IS $\begin{pmatrix} -1/5 & 1/10 & 2/5 \\ 2/5 & 3/10 & -4/5 \\ 0 & -1/2 & 1 \end{pmatrix}$.

Continued ⟶

(b) Recall that each elementary row operation is reversible. Go back to each of your matrices from above and determine the elementary matrix that reverses (undoes) each row operation. Use the notation E_j' for the elementary matrix that "undoes" E_j.

$$E_1' = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_2' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}, \quad E_3' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_4' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{pmatrix}, \quad E_5' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 4/5 \\ 0 & 0 & 1 \end{pmatrix}, \quad E_6' = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_7' = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c) Use a computer or calculator to check that $E_1'E_2'\cdots E_k' = A$.

$$E_1' E_2' E_3' E_4' E_5' E_6' E_7' = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 2 & 0 \\ 2 & 1 & 1 \end{pmatrix}$$

Congratulations! You have just shown, by example, that every nonsingular matrix is a product of elementary matrices. This will be a very useful idea in the future.