Math 236 - Assignment 5

February 25, 2026

Name ______________________

Score __________

Show all work to receive full credit. Supply explanations when necessary. You may use technology to solve any linear systems. This assignment is due March 4.

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1. Consider the matrix $M = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ -1 & 2 & 7 \end{pmatrix}$.
   (a) Determine whether the row $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ is in the row space of M.
   (b) Find a basis for the row space of M.
   (c) Find the representation for the row $\begin{pmatrix} -3 & 8 & 27 \end{pmatrix}$ in terms of your basis.
2. Argue that the rank of a matrix is equal to the rank of its transpose.
3. Give an example to show that the column space of a matrix and the row space of the matrix are, in general, not the same even though they have the same dimension.
4. Suppose that $A \in \mathcal{M}_{m\times n}$. Argue that the rank of A is less than or equal to $\min\{m, n\}$.
5. Describe all matrices that have rank 0. Then find a general description for all matrices of rank 1.
6. Consider the function $F : \mathbb{R}^4 \to \mathcal{M}_{2\times 2}$ defined by

$$F(\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}) = \begin{pmatrix} c & a+d \\ b & d \end{pmatrix}.$$

   Show that F is one-to-one and onto.
7. Define three different isomorphisms between $\mathbb{R}^3$ and $\mathcal{P}_2$. You don't need to prove that they are actually isomorphisms (just be sure of it).
8. Explain why an isomorphism between $\mathbb{R}^3$ and $\mathcal{M}_{2\times 2}$ does not exist.
9. Show that the map $F : \mathcal{P}_2 \to \mathcal{P}_2$ given by $F(ax^2 + bx + c) = bx^2 - (a + c)x + a$ is an isomorphism.
10. For an arbitrary 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant of A is defined by $\det(A) = ad - bc$. Show that the determinant function is not an isomorphism from $\mathcal{M}_{2\times 2}$ into $\mathbb{R}$.
11. Suppose that $h : V \to W$ is a homomorphism and that $\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\}$ is a linearly dependent set in V. Prove that $\{h(\vec{v_1}), h(\vec{v_2}), \ldots, h(\vec{v_n})\}$ is a linearly dependent set in W.