Math 236 - Assignment 3

February 4, 2026

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations when necessary. Do all computations by hand unless otherwise indicated. This assignment is due February 11.

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1. Let V be the set of all 2×2 nonsingular matrices with the usual operations of matrix addition and scalar multiplication. Show that V is NOT a vector space.

2. Show that P is a vector space with the usual operations of polynomial addition and multiplication by a constant.

$$P = \{p \in \mathcal{P}_2 : p(x) = p(-x) \text{ for all } x \in \mathbb{R}\}.$$

   (It might be helpful to start by determining a description for the polynomials in P.)

3. Let V be the set of all vectors in $\mathbb{R}^2$ with the usual addition. However, define scalar multiplication '$\cdot$' in V as follows:

$$a \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ a^2 y \end{pmatrix}.$$

   Show that V is NOT a vector space.

4. Show that the set $\mathbb{R}^+$ of positive real numbers is a vector space when we interpret the "sum", $x + y$, as the product of x and y, and we interpret scalar "multiplication", $k \cdot x$, as the kth power of x.

5. Prove that in a vector space, the zero vector is unique. Use only the ten vector space conditions. (Hint: We often prove the uniqueness of a mathematical object by assuming there are two objects with the given property, and then concluding that the objects must be the same.)

6. Prove that in a vector space, if $a\vec{v} = \vec{0}$, then $a = 0$ or $\vec{v} = \vec{0}$. Use only the ten vector space conditions and/or Lemma 1.16. (Hint: You may use the fact that in any field of scalars, any nonzero scalar has a multiplicative inverse. If you need more of a hint, just ask.)

7. Is $W = \{p \in \mathcal{P}_2 : p(1) = 1\}$ a subspace of $\mathcal{P}_2$?

8. Determine if $\begin{pmatrix} 0 & 1 \\ 4 & 2 \end{pmatrix}$ is in the span of $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 2 & 0 \\ 2 & 3 \end{pmatrix}$. What about $\begin{pmatrix} -5 & 0 \\ -5 & -12 \end{pmatrix}$?

9. Show that R is a subspace of $\mathcal{P}_2$.

$$R = \{p \in \mathcal{P}_2 : p(2) = 0\}$$

   Then parameterize the subspace's description, and express the subspace as a span of vectors in $\mathcal{P}_2$.