MTH 236 Assignment 2 Key

1) Every person has their own blood type,
so the relation is obviously reflexive.

To show symmetric,
Assume Person 1 has the same blood type as Person 2. So, if Person 1 has blood type Q, then Person 2, has blood type Q. Therefore, Person 2 has the same blood type as Person 1.

To show transitive,
Assume Person 1 has the same blood type as Person 2 and Person 2 has the same blood type as Person 3. Say Person 1 has blood type Q. Then Person 2 has blood type Q, and then so must Person 3. So all three has blood type Q, and therefore Person 1 has the same blood type as Person 3.

2) WE ARE SAYING $m \sim n$ IFF $m-n = 7p$ FOR SOME INTEGER p.

REFLEXIVE: FOR ANY INTEGER m, $m-m = 0 = 7(0)$, SO $m-m$ IS A MULTIPLE OF 7. THEREFORE, $m \sim m$.

SYMMETRIC: SUPPOSE m & n ARE INTEGERS WITH $m \sim n$. THEN $m-n = 7p$, AND IT FOLLOWS THAT $n-m = -7p$. SO $n-m$ IS A MULTIPLE OF 7. THEREFORE, $n \sim m$.

TRANSITIVE: SUPPOSE $m, n,$ & k ARE INTEGERS WITH $m \sim n$ AND $n \sim k$. THEN $m-n = 7p$ AND $n-k = 7q$. BY ADDING THESE EQUATIONS, WE FIND $m-k = 7(p+q)$. SO $m-k$ IS A MULTIPLE OF 7. THEREFORE, $m \sim k$.

3) WE ARE SAYING $p \sim q$ IFF $p = kq$ FOR SOME INTEGER k.

THIS RELATION IS NOT SYMMETRIC.

IF $p = kq$ FOR SOME NONZERO INTEGER k,

THEN $q = \frac{1}{k} p$ AND $\frac{1}{k}$ IS

NOT AN INTEGER.

$p \sim q$ DOES NOT IMPLY $q \sim p$.

4) THREE LINEAR COMBOS OF $y_1(x) = \cos x$ AND $y_2(x) = \sin x$ ARE

$$w_1(x) = a \cos x + b \sin x$$

$$w_2(x) = c \cos x + d \sin x$$

$$w_3(x) = e \cos x + f \sin x$$

A LINEAR COMBO OF w_1, w_2, w_3 IS

$$z(x) = \alpha(a \cos x + b \sin x) + \beta(c \cos x + d \sin x) + \gamma(e \cos x + f \sin x)$$

$$= \cdots = (\alpha a + \beta c + \gamma e) \cos x + (\alpha b + \beta d + \gamma f) \sin x$$

AND z, IS A LINEAR COMBO OF

$y_1(x) = \cos x$ AND $y_2(x) = \sin x$. ▨

5) $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 4 & 4 \\ 2 & 3 & 4 & 9 & 16 \\ -2 & 0 & 3 & -7 & 11 \end{array}\right)$

$R_2 = -2R_1 + R_2$

$R_3 = 2R_1 + R_3$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 4 & 4 \\ 0 & 1 & 2 & 1 & 8 \\ 0 & 2 & 5 & 1 & 19 \end{array}\right)$$

$R_3 = -2R_2 + R_3$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 4 & 4 \\ 0 & 1 & 2 & 1 & 8 \\ 0 & 0 & 1 & -1 & 3 \end{array}\right)$$

$R_2 = -2R_3 + R_2$

$R_1 = -R_3 + R_1$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 5 & 1 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -1 & 3 \end{array}\right)$$

$R_1 = -R_2 + R_1$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -1 & 3 \end{array}\right)$$

RREF

6)
$$\left(\begin{array}{ccccc|c} 2 & -3 & 1 & -3 & 2 & 6 \\ 2 & -3 & 5 & -1 & 1 & 8 \\ 0 & 0 & 4 & 3 & 2 & 3 \\ -2 & 3 & 3 & 3 & -9 & -6 \end{array}\right)$$

$$\text{RREF} \longrightarrow \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \text{rhs} \end{array}$$
$$\left(\begin{array}{ccccc|c} 1 & -3/2 & 0 & 0 & 5/8 & 9/2 \\ 0 & 0 & 1 & 0 & -7/4 & 0 \\ 0 & 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

LEADING: x_1, x_3, x_4 FREE: x_2, x_5

SOLUTION SET:

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 9/2 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} -5/8 \\ 0 \\ 7/4 \\ -3 \\ 1 \end{pmatrix} x_5; \quad x_2, x_5 \in \mathbb{R}}$$

7) THE MATRIX $\begin{pmatrix} 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$ IS NOT IN

RREF FOR TWO REASONS:

① THE 1,1 ENTRY SHOULD BE A 1

② THE 1,3 ENTRY SHOULD BE A 0.

$$R_1 = -3R_2 + R_1 \longrightarrow \begin{pmatrix} 2 & 0 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

$$R_1 = \tfrac{1}{2}R_1 \longrightarrow \begin{pmatrix} 1 & 0 & 0 & -5/2 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

RREF

8)

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 0 \\ 3 & -1 & 0 \end{pmatrix} \xrightarrow{\substack{R_3^{(2)} = -3R_1^{(1)} + R_3^{(1)} \\ R_2^{(2)} = \frac{1}{4}R_2^{(1)}}} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -7 & -3 \end{pmatrix}$$

$$\xrightarrow{R_3^{(3)} = 7R_2^{(2)} + R_3^{(2)}} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$\xrightarrow{\substack{R_3^{(4)} = -\frac{1}{3}R_3^{(3)} \\ R_1^{(2)} = -R_3^{(4)} + R_1^{(1)}}} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_1^{(3)} = -2R_2^{(2)} + R_1^{(2)}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_3^{(4)} = -\frac{1}{3}\left(7R_2^{(2)} + R_3^{(2)}\right) = -\frac{1}{3}\left(7\left(\frac{1}{4}R_2^{(1)}\right) + \left(-3R_1^{(1)} + R_3^{(1)}\right)\right)$$

$$= R_1^{(1)} - \frac{7}{12}R_2^{(1)} - \frac{1}{3}R_3^{(1)}$$

$$\boxed{R_3^{(4)} = R_1^{(1)} - \frac{7}{12}R_2^{(1)} - \frac{1}{3}R_3^{(1)}}$$

OR

$$\boxed{(0, 0, 1) = (1, 2, 1) - \frac{7}{12}(0, 4, 0) - \frac{1}{3}(3, -1, 0)}$$

9) Let s = cost of sandwich

c = cost of coffee

d = cost of doughnut

$$4s + c + 10d = 8.45$$
$$3s + c + 7d = 6.30$$

Our goal is to find $s + c + d$.

$$\left(\begin{array}{ccc|c} 4 & 1 & 10 & 8.45 \\ 3 & 1 & 7 & 6.30 \end{array}\right)$$

$$\text{RREF} \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 3 & 2.15 \\ 0 & 1 & -2 & -0.15 \end{array}\right)$$

$$R_2 = R_1 + R_2 \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 3 & 2.15 \\ 1 & 1 & 1 & 2.00 \end{array}\right)$$

$$\boxed{s + c + d = 2.00}$$