Math 236 - Assignment 2

Name _____

January 29, 2025

Score _____

Show all work to receive full credit. Supply explanations when necessary. Do all computations by hand unless otherwise indicated. This assignment is due February 5.

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1. For any real numbers x and y, we will say $x \sim y$ if and only if $x - y$ is an integer. Prove that $\sim$ is an equivalence relation.

2. Prove that a linear combination of three linear combinations of x, y, and z is a linear combination of x, y, and z.

3. Find (by hand) the reduced row echelon form (RREF).

$$\begin{pmatrix} 1 & 0 & 3 & 1 & 2 \\ 1 & 4 & 2 & 1 & 5 \\ 3 & 4 & 8 & 1 & 2 \end{pmatrix}$$

4. Write the system as an augmented matrix. Then compute the RREF and determine the solution set. You may use technology to find the RREF.

$$\begin{array}{rcrcrcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & - & x_5 & = & 1 \\ 3x_1 & - & x_2 & + & x_3 & + & x_4 & + & x_5 & = & 3 \end{array}$$

5. Say why the following matrix is NOT in RREF. Then reduce it to RREF. Finally, once it's reduced, show that no row is a linear combination of the other rows.

$$\begin{pmatrix} 1 & 2 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

6. Give two distinct echelon forms of the matrix below. Be sure to say (or show) which sequence of row operations gave each form.

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 6 & 4 & 1 & 2 \\ 1 & 5 & 1 & 5 \end{pmatrix}$$

7. Reduce to RREF. Then express the third row of the RREF as a linear combination of the rows of the original matrix.

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 0 \\ 3 & -1 & 0 \end{pmatrix}$$

8. Three truck drivers (in perhaps 1960) went into a roadside cafe. One truck driver purchased four sandwiches, a cup of coffee, and ten doughnuts for \$8.45. Another driver purchased three sandwiches, a cup of coffee, and seven doughnuts for \$6.30. What did the third truck driver pay for a sandwich, a cup of coffee, and a doughnut? (If your approach to this problem involves matrices and elimination, you may use technology.)