Math 233 - Quiz 3

February 5, 2026

Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (7 points) Consider the triangle with vertices at the points $A(2, 4, -2)$, $B(1, 1, 3)$, and $C(-3, 3, -1)$.

   (a) Find the area of $\triangle ABC$.

$$\vec{AB} = -\hat{i} - 3\hat{j} + 5\hat{k}$$
$$\vec{AC} = -5\hat{i} - \hat{j} + \hat{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 5 \\ -5 & -1 & 1 \end{vmatrix} = 2\hat{i} - 24\hat{j} - 14\hat{k} = 2(\hat{i} - 12\hat{j} - 7\hat{k})$$

$$\|\vec{AB} \times \vec{AC}\| = 2\sqrt{1 + 144 + 49}$$
$$= 2\sqrt{194}$$

$$\text{Area} = \frac{1}{2}\|\vec{AB} \times \vec{AC}\|$$
$$= \boxed{\sqrt{194}}$$
$$\approx 13.9$$

   (b) Find an equation of the plane passing through the points A, B, and C.

$$\vec{n} = \vec{AB} \times \vec{AC} = 2\hat{i} - 24\hat{j} - 14\hat{k}$$

I'll instead use

$$\vec{n} = \hat{i} - 12\hat{j} - 7\hat{k}$$

Using $(1,1,3)$
Plane is

$$x - 12y - 7z = 1 - 12 - 21 = -32$$

$$\boxed{x - 12y - 7z = -32}$$

   (c) Find parametric and symmetric equations of the line through points A and B.

$$\vec{AB} = -\hat{i} - 3\hat{j} + 5\hat{k}$$

Using $A(2, 4, -2)$

PARAMETRIC:
$$x = 2 - t$$
$$y = 4 - 3t$$
$$z = -2 + 5t$$

Symmetric:
$$\frac{x-2}{-1} = \frac{y-4}{-3} = \frac{z+2}{5}$$

2. (3 points) Find the measure of the angle between the planes $3x - 2y + z = 4$ and $x + 8y + 2z = 0$. Write your final answer in degrees rounded to the nearest tenth.

$$\vec{n}_1 = 3\hat{i} - 2\hat{j} + \hat{k}$$
$$\vec{n}_2 = \hat{i} + 8\hat{j} + 2\hat{k}$$

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{|-11|}{\sqrt{14}\sqrt{69}} = \frac{11}{\sqrt{14}\sqrt{69}}$$

$$\theta = \cos^{-1}\left(\frac{11}{\sqrt{14}\sqrt{69}}\right) \approx \boxed{69.3°}$$