Math 233 - Quiz 2

January 29, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) P and Q are the points $(4, -1, 6)$ and $(-2, -3, 5)$, respectively. Find a vector of length 8 that is parallel to $\vec{PQ}$. How many different answers are possible? Briefly explain.

$\vec{PQ} = \langle -2-4, -3-(-1), 5-6 \rangle$

$= \langle -6, -2, -1 \rangle$

$\|\vec{PQ}\| = \sqrt{36+4+1}$

$= \sqrt{41}$

THERE ARE TWO POSSIBLE ANSWERS:
ONE IN THE DIRECTION OF $\vec{PQ}$
AND ONE IN THE OPPOSITE DIRECTION.

$$\frac{\pm 8\vec{PQ}}{\|\vec{PQ}\|} = \pm\left(\frac{-48}{\sqrt{41}}\hat{i} - \frac{16}{\sqrt{41}}\hat{j} - \frac{8}{\sqrt{41}}\hat{k}\right)$$

2. (3 points) Determine the measure of the angle between the vectors $\vec{x} = 4\hat{i} - 7\hat{j} - 2\hat{k}$ and $\vec{y} = 5\hat{i} + 2\hat{j} + 8\hat{k}$. Write your final answer in degrees, rounded to the nearest hundredth.

LET'S USE $\vec{x} \cdot \vec{y} = \|\vec{x}\| \, \|\vec{y}\| \cos\theta$

$\vec{x} \cdot \vec{y} = 20 - 14 - 16 = -10$

$\|\vec{x}\| = \sqrt{16+49+4} = \sqrt{69}$

$\|\vec{y}\| = \sqrt{25+4+64} = \sqrt{93}$

$$\cos\theta = \frac{-10}{\sqrt{69}\sqrt{93}}$$

$$\theta = \cos^{-1}\left(\frac{-10}{\sqrt{69}\sqrt{93}}\right) \approx 97.17^\circ$$

3. (2 points) Compute the projection of $\vec{w} = \hat{i} + 3\hat{j} - 5\hat{k}$ onto $\vec{z} = -5\hat{i} - \hat{j} + 6\hat{k}$.

$$\text{proj}_{\vec{z}}\,\vec{w} = \frac{\vec{w} \cdot \vec{z}}{\vec{z} \cdot \vec{z}}\,\vec{z}$$

$\vec{w} \cdot \vec{z} = -5 - 3 - 30 = -38$

$\vec{z} \cdot \vec{z} = 25 + 1 + 36 = 62$

$$\text{proj}_{\vec{z}}\,\vec{w} = \frac{-38}{62}\vec{z} = \frac{-19}{31}\vec{z}$$

$$= \frac{95}{31}\hat{i} + \frac{19}{31}\hat{j} - \frac{114}{31}\hat{k}$$

4. (2 points) What does it mean for two vectors $\vec{a}$ and $\vec{b}$ to be parallel? What does it mean for two vectors $\vec{p}$ and $\vec{q}$ to be orthogonal?

$\vec{a}$ IS PARALLEL TO $\vec{b}$ MEANS $\vec{a}$ IS A NONZERO SCALAR MULTIPLE OF $\vec{b}$.

$(\vec{a} = k\vec{b})$

$\vec{p}$ IS ORTHOGONAL TO $\vec{q}$ MEANS $\vec{p} \cdot \vec{q} = 0$.