Math 233 - Quiz 4

Name key

September 17, 2026

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3.5 points) Find the function $\vec{r}(t)$ that satisfies

$$\vec{r}\,'(t) = e^{-2t}\,\hat{\imath} - t\,\hat{\jmath} + \frac{5t}{t^2+1}\,\hat{k}; \qquad \vec{r}(0) = 5\hat{\imath} + 3\hat{\jmath} - 2\hat{k}.$$

$$\vec{r}(t) = \hat{\imath}\int e^{-2t}\,dt - \hat{\jmath}\int t\,dt + \hat{k}\int \frac{5t}{t^2+1}\,dt$$

$u = t^2+1$

$\frac{1}{2}\,du = t\,dt$

$$\vec{r}(0) = 5\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$$

$$\Rightarrow -\tfrac{1}{2} + C_1 = 5 \qquad C_1 = \tfrac{11}{2}$$

$$C_2 = 3$$

$$C_3 = -2$$

$$\vec{r}(t) = \left(-\tfrac{1}{2}e^{-2t} + C_1\right)\hat{\imath} + \left(-\tfrac{1}{2}t^2 + C_2\right)\hat{\jmath} + \left[\tfrac{5}{2}\ln(t^2+1) + C_3\right]\hat{k}$$

$$\boxed{\vec{r}(t) = \left(-\tfrac{1}{2}e^{-2t} + \tfrac{11}{2}\right)\hat{\imath} + \left(-\tfrac{1}{2}t^2 + 3\right)\hat{\jmath} + \left[\tfrac{5}{2}\ln(t^2+1) - 2\right]\hat{k}}$$

2. (3 points) Let $\vec{r}(t) = \sin 6t\,\hat{\imath} + \cos 6t\,\hat{\jmath} + 8t\,\hat{k}$. Compute $\hat{T}(t)$.

$$\vec{r}\,'(t) = 6\cos 6t\,\hat{\imath} - 6\sin 6t\,\hat{\jmath} + 8\hat{k}$$

$$\|\vec{r}\,'(t)\| = \sqrt{36\cos^2 6t + 36\sin^2 6t + 64}$$

$$= \sqrt{36+64} = 10$$

$$\boxed{\hat{T}(t) = \frac{3\cos 6t\,\hat{\imath} - 3\sin 6t\,\hat{\jmath} + 4\hat{k}}{5}}$$

3. (2 points) Refer to the function $\vec{r}(t)$ in problem 2. Without actually computing the dot products, would you expect $\vec{r}(t)\cdot\vec{r}\,'(t) = 0$? Would you expect $\vec{r}\,'(t)\cdot\vec{r}\,''(t) = 0$? Briefly explain.

$\vec{r}$ DOES NOT HAVE CONSTANT MAG. I WOULD NOT EXPECT $\vec{r}\cdot\vec{r}\,' = 0$.

$\vec{r}\,'$ DOES HAVE CONSTANT MAG. $\vec{r}\,'\cdot\vec{r}\,'' = 0$

4. (1.5 points) What are the three important characteristics of the principal unit tangent vector.

① $\hat{T}$ POINTS IN THE DIRECTION OF THE CURVE'S ORIENTATION

② $\hat{T}$ IS A UNIT VECTOR: $\|\hat{T}(t)\| = 1$

③ $\hat{T}$ IS TANGENT TO THE GRAPH OF $\vec{r}$ AT t.