Math 233 - Quiz 11

November 20, 2025

Name key

Score ______

Show all work to receive full credit. Supply explanations when necessary. Use other paper as necessary. This quiz is due December 2.

1. (2 points) Use Lagrange multipliers to find the extreme values of $f(x,y) = y^2 - 4x$ subject to $x^2 + y^2 = 9$.

$\vec{\nabla} f(x,y) = -4\hat{\imath} + 2y\hat{\jmath}$

$\vec{\nabla} g(x,y) = 2x\hat{\imath} + 2y\hat{\jmath}$

$-4 = \lambda 2x$, $2y = \lambda 2y$, $x^2+y^2=9$

$2y - \lambda 2y = 0$

$\Downarrow$

$\lambda = 1$ OR $y = 0$

$x = -2$, $y = \pm\sqrt{5}$; $x = \pm 3$

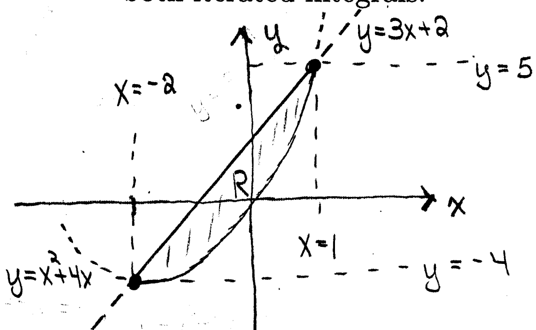
"CRIT PTS" ARE $(3,0)$, $(-3,0)$, $(-2,\sqrt{5})$, $(-2,-\sqrt{5})$

$\boxed{f(3,0) = -12 \leftarrow \text{MIN}}$

$f(-3,0) = 12$

$\left.\begin{array}{l} f(-2,\sqrt{5}) = 13 \\ f(-2,-\sqrt{5}) = 13 \end{array}\right\}$ MAX.

2. (3 points) Sketch the region of integration, reverse the order of integration, and evaluate both iterated integrals.



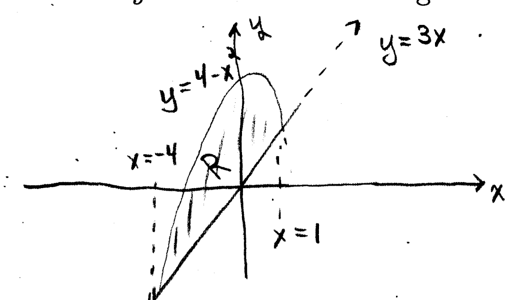
$$\int_{-2}^{1}\int_{x^2+4x}^{3x+2} dy\,dx = \int_{y=-4}^{y=5}\int_{x=\frac{y-2}{3}}^{x=\sqrt{y+4}-2} dx\,dy$$

$y = (x+2)^2 - 4$

$x = \sqrt{y+4} - 2$

SEE ATTACHED SHEET. (PAGE 2)

3. (2.5 points) Let E be the plane region in the xy-plane between the graphs of $y = 4 - x^2$ and $y = 3x$. Sketch the region E and evaluate the double integral $\iint (x+4)\,dA$.



$4 - x^2 = 3x$

$\Rightarrow x^2 + 3x - 4 = 0$

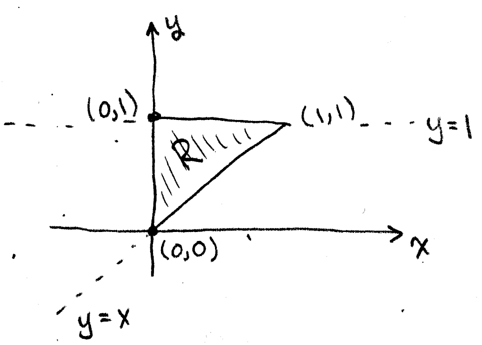
$(x+4)(x-1) = 0$

$x = -4$, $x = 1$

$$\int_{x=-4}^{x=1}\int_{y=3x}^{y=4-x^2} (x+4)\,dy\,dx = \int_{-4}^{1} (x+4)(4-x^2-3x)\,dx$$

SEE ATTACHED SHEET. (PAGE 3)

4. (2.5 points) Use a double integral to find the volume of the space region under the graph of $z = x + y + 2$ and over the triangle in the xy-plane with vertices at $(0,0)$, $(0,1)$, and $(1,1)$.



$$\int_{x=0}^{x=1}\int_{y=x}^{y=1} (x+y+2)\,dy\,dx = \int_0^1 \left(xy + \tfrac{1}{2}y^2 + 2y\right)\Big|_{y=x}^{y=1} dx$$

$$= \int_0^1 \left(x + \tfrac{1}{2} + 2 - x^2 - \tfrac{1}{2}x^2 - 2x\right) dx = \int_0^1 \left(-\tfrac{3}{2}x^2 - x + \tfrac{5}{2}\right) dx$$

$$= -\tfrac{1}{2} - \tfrac{1}{2} + \tfrac{5}{2} = \boxed{\tfrac{3}{2}}$$

2.) CONTINUED...

$$\int_{-2}^{1}\int_{x^2+4x}^{3x+2} dy\,dx = \int_{-2}^{1}\left(3x+2-x^2-4x\right)dx$$

$$-\int_{-2}^{1}\left(x^2+x-2\right)dx = \frac{1}{3}x^3+\frac{1}{2}x^2-2x \Big|_{1}^{-2}$$

$$= \left[\frac{-8}{3}+2+4\right]-\left[\frac{1}{3}+\frac{1}{2}-2\right]$$

$$= \boxed{\frac{9}{2}}$$

$$\int_{-4}^{5}\int_{\frac{y-2}{3}}^{\sqrt{y+4}-2} dx\,dy = \int_{-4}^{5}\left(\sqrt{y+4}-2-\frac{1}{3}y+\frac{2}{3}\right)dy$$

$$= \frac{2}{3}(y+4)^{3/2}-\frac{4}{3}y-\frac{1}{6}y^2 \Big|_{-4}^{5}$$

$$= \left[18-\frac{20}{3}-\frac{25}{6}\right]-\left[0+\frac{16}{3}-\frac{16}{6}\right]$$

$$= 18-\frac{81}{6} = \frac{27}{6} = \boxed{\frac{9}{2}}$$

3.) CONTINUED ...

$$\int_{-4}^{1} \left(4x - x^3 - 3x^2 + 16 - 4x^2 - 12x\right) dx = \int_{-4}^{1} \left(-x^3 - 7x^2 - 8x + 16\right) dx$$

$$= -\frac{1}{4}x^4 - \frac{7}{3}x^3 - 4x^2 + 16x \Big|_{-4}^{1}$$

$$= \left(-\frac{1}{4} - \frac{7}{3} - 4 + 16\right) - \left(-\frac{256}{4} + \frac{448}{3} - 64 - 64\right)$$

$$= \boxed{\frac{625}{12}}$$