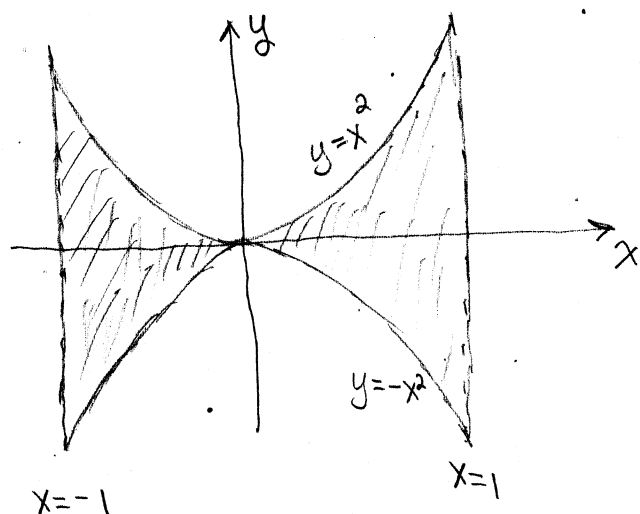


(1)

MTH 233 Assignment 10 key

$$1) \int_{-1}^1 \int_{-x^2}^{x^2} (x^2 - y) dy dx = \int_{-1}^1 \left. x^2 y - \frac{1}{2} y^2 \right|_{y=-x^2}^{y=x^2} dx$$

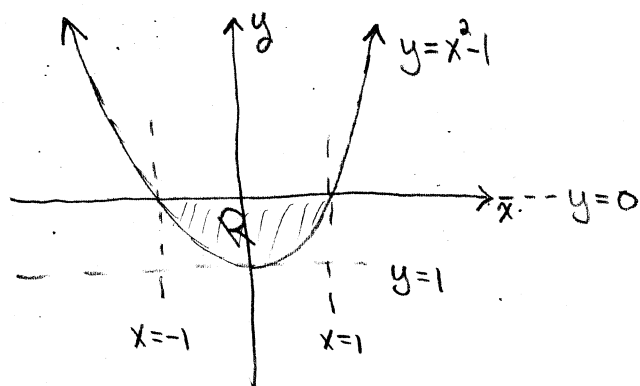


$$= \int_{-1}^1 \left(x^4 - \frac{1}{2} x^4 \right) - \left(-x^4 - \frac{1}{2} x^4 \right) dx$$

$$= \int_{-1}^1 2x^4 dx = \left. \frac{2}{5} x^5 \right|_{-1}^1$$

$$= \frac{2}{5} + \frac{2}{5} = \boxed{\frac{4}{5}}$$

$$2) \int_{-1}^0 \int_{-\sqrt{y+1}}^{\sqrt{y+1}} y^2 dx dy = \int_{x=-1}^{x=1} \int_{y=x^2-1}^{y=0} y^2 dy dx$$



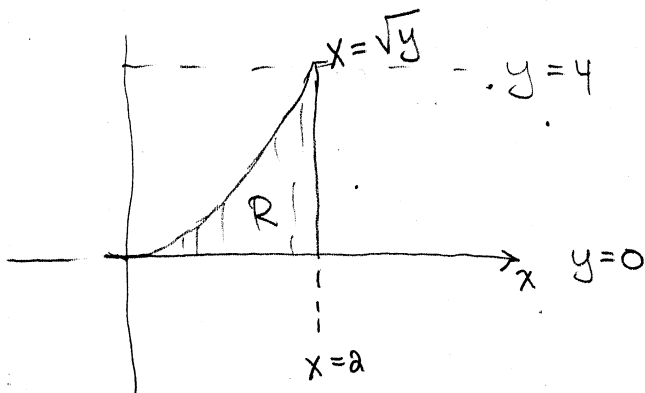
$$= \int_{-1}^1 \left. \frac{1}{3} y^3 \right|_{y=x^2-1}^{y=0} dx$$

$$= - \int_{-1}^1 \frac{1}{3} (x^2 - 1)^3 dx = - \frac{2}{3} \int_0^1 (x^2 - 1)^3 dx$$

$$= - \frac{2}{3} \int_0^1 (x^6 - 3x^4 + 3x^2 - 1) dx$$

$$= - \frac{2}{3} \left(\frac{1}{7} - \frac{3}{5} + 1 - 1 \right) = \boxed{\frac{32}{105}}$$

$$3) \int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy = \int_{x=0}^{x=2} \int_{y=0}^{y=x^2} e^{x^3} dy dx$$



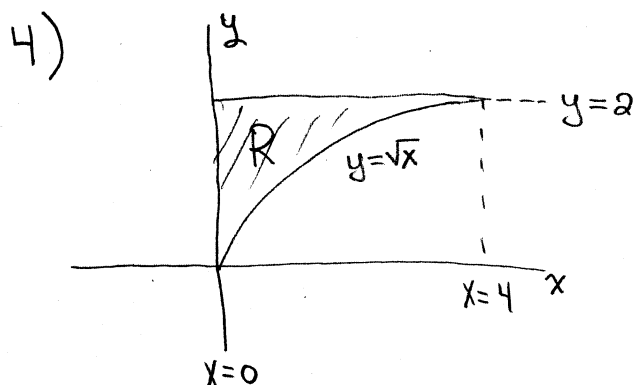
$$= \int_0^2 x^2 e^{x^3} dx$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{3} \int_0^8 e^u du = \boxed{\frac{1}{3} e^8 - \frac{1}{3}}$$



$$\iint_R \sin y^3 dA$$

$$= \int_{x=0}^{x=4} \int_{y=\sqrt{x}}^{y=2} \sin y^3 dy dx$$

$$= \int_{y=0}^{y=2} \int_{x=0}^{x=y^2} \sin y^3 dx dy$$

BOTH
ORDERS.

$$\left. \frac{1}{3} \cos u \right|_{u=8}^{u=0}$$

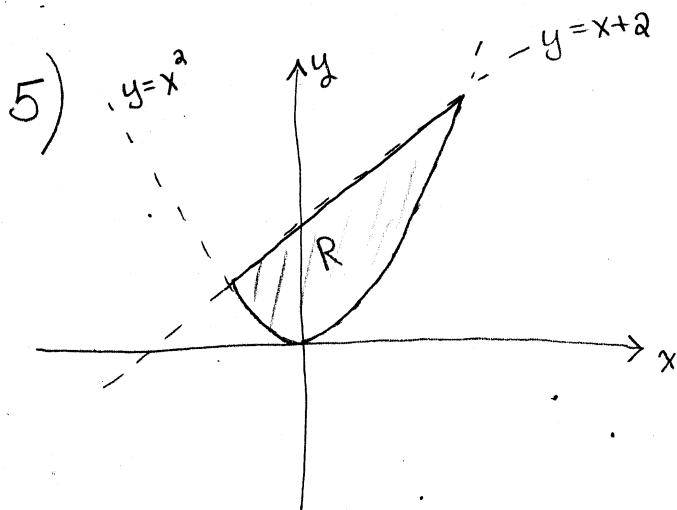
$$= \frac{1}{3} - \frac{1}{3} \cos(8) \approx 0.3818$$

$$= \int_0^2 y^2 \sin y^3 dy = \int_0^8 \frac{1}{3} \sin u du$$

$$u = y^3$$

$$du = 3y^2 dy$$

$$\frac{1}{3} du = y^2 dy$$



$$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0 \Rightarrow x = -1, x = 2$$

$$\iint_R (xy+5) dA$$

$$= \int_{x=-1}^2 \int_{y=x^2}^{y=x+2} (xy+5) dy dx$$

$$= \int_{-1}^2 \left. \frac{xy^2}{2} + 5y \right|_{y=x^2}^{y=x+2} dx$$

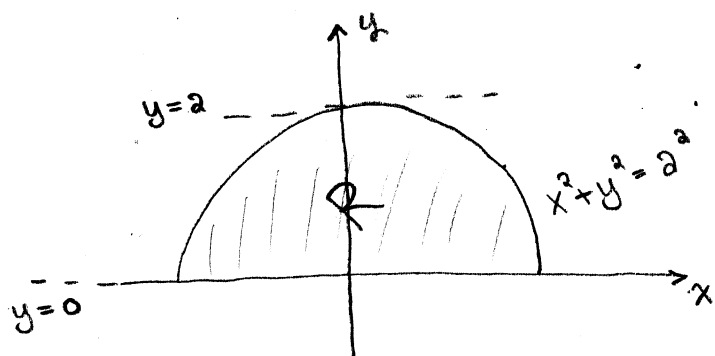
$$= \int_{-1}^2 \left(\frac{1}{2}x^3 + 2x^2 + 2x + 5x + 10 - \frac{x^5}{2} - 5x^2 \right) dx$$

$$= \int_{-1}^2 \left(-\frac{1}{2}x^5 + \frac{1}{2}x^3 - 3x^2 + 7x + 10 \right) dx$$

$$= \left. -\frac{1}{12}x^6 + \frac{1}{8}x^4 - x^3 + \frac{7}{2}x^2 + 10x \right|_{-1}^2$$

$$= \frac{68}{3} + \frac{131}{24} = \boxed{\frac{225}{8}}$$

6)
$$\int_0^a \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (x^2+y^2)^2 dx dy$$



$$\int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=2} (r^2)^2 r dr d\theta = \pi \int_0^2 r^5 dr = \frac{\pi}{6} r^6 \Big|_0^2 = \boxed{\frac{32\pi}{3}}$$