Math 233 - Final Exam B   Name _____

December 15, 2022   Score _____

Show all work to receive full credit. Supply explanations where necessary. Place your final answer in the box provided.

1. (5 points) The points A, B, and C are the vertices of a triangle in space. Find the measure of the angle at B. Write your answer in degrees, rounded to the nearest hundredth.

$$A\,(0, -3, 7), \qquad B\,(1, -2, 3), \qquad C\,(5, 1, -2)$$

<div style="border:1px solid black; height:150px;"></div>

2. (5 points) A parallelepiped is determined by the vectors $\vec{u} = 3\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$, $\vec{v} = 2\hat{\imath} + \hat{\jmath} - 5\hat{k}$, and $\vec{w} = -3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$. Find the area of the face that is the parallelogram determined by $\vec{u}$ and $\vec{w}$.

<div style="border:1px solid black; height:150px;"></div>

1

3. (5 points) Find symmetric equations for the line through $P\,(17, 9, -11)$ and $Q\,(12, -1, -10)$.

4. (5 points) An object is moved along the graph of

$$\vec{r}(t) = t^2\,\hat{\imath} + \cos(\pi t)\,\hat{\jmath} + \sqrt{t}\,\hat{k}$$

from the point $(0, 1, 0)$ to the point $(16, 1, 2)$. Set up the definite integral that gives the length of the path. Use you calculator to approximate the value of the integral.

5. (5 points) Ralphie shoots his Red Ryder BB gun from $4\,\text{ft}$ above the ground across a level field while holding the barrel of his BB gun at an angle of $5°$ above horizontal. Assuming the BB's muzzle velocity is $350\,\text{ft/s}$, how far downrange will the BB travel before hitting the ground?

6. (5 points) Find the limit or show that it does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^3}{x^2 + y^2}$

7. (5 points) Find the limit or show that it does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

8. (5 points) The period, T, of a pendulum of length L is given by

$$T = \frac{2\pi\sqrt{L}}{\sqrt{g}},$$

where g is the acceleration due to gravity. A pendulum is moved from a location where $g = 32.09\,\text{ft/s}^2$ to a location where $g = 32.23\,\text{ft/s}^2$. There was also a temperature change that resulted in a change in the length of the pendulum from $2.5\,\text{ft}$ to $2.48\,\text{ft}$. Use differentials to approximate the corresponding change in the pendulum's period.

9. (5 points) Find an equation of the plane tangent to the surface $ze^{x^2-y^2} - 3 = 0$ at the point $P(2, 2, 3)$.

10. (5 points) Find the directional derivative of

$$f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$$

at the point $(5, -5, 5)$ in the direction the function decreases most rapidly.

11. (5 points) Let D be the region in the second quadrant bounded by graphs of $y = \frac{1}{2}x^2$, $y = 2$, and $x = 0$. Write the double integral as an iterated integral and evaluate.

$$\iint\limits_{D} x^5 \sin(y^4)\, dA$$

12. (5 points) Sketch the region of integration. Then reverse the order of integration and evaluate.

$$\int_0^8 \int_{\sqrt[3]{y}}^2 \frac{y}{x^7 + 1}\, dx\, dy$$

13. (5 points) Convert the iterated integral to an equivalent integral in cylindrical coordinates. DO NOT EVALUATE.

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_{-\sqrt{2x^2+2y^2}}^{6+x^2+y^2} 15z \, dz \, dy \, dx$$

14. (5 points) Evaluate

$$\int_C y \, dx + x^2 \, dy,$$

where C is the parabolic arc along $y = 4x - x^2$ from $(4, 0)$ to $(1, 3)$.