

Math 233 - Final Exam A

December 9, 2022

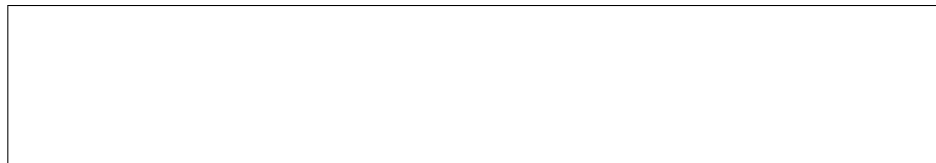
Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Place your final answer in the box provided. This test is due December 15. You must work individually.

1. (5 points) Use a double integral to find the area of the 1st quadrant region inside both the circle $r = 2 \cos \theta$ and the circle $r = 4 \sin \theta$. After sketching the region and setting up the required iterated integral(s), you may use technology to evaluate the integral(s). Round your final answer to four decimal places.

2. (5 points) A solid lies in the 1st octant bounded by the coordinates planes, the plane $y + z = 2$, and the parabolic cylinder $x = 4 - y^2$. The density of the solid at the point (x, y, z) is given by $\rho(x, y, z) = 2 + z + \sin x$. Set up the triple integral that gives the mass of the solid. Use technology to evaluate the integral. Round your final answer to four decimal places.



3. (10 points) Let $\vec{F}(x, y) = (y^3 + 1)\hat{i} + (3xy^2 + 1)\hat{j}$.

- (a) Use our test to show that \vec{F} is a conservative vector field. Then find a scalar potential function for \vec{F} . Write the potential function in the box below.

- (b) Evaluate $\int_C \vec{F}(x, y) \cdot d\vec{r}$ where C is the semicircular path in the 1st quadrant from $(0, 0)$ to $(2, 0)$. (Hint: Use the fact that \vec{F} is conservative.)

4. (5 points) Use Green's Theorem to evaluate

$$\int_C \cos y \, dx + (xy - x \sin y) \, dy,$$

where C is the positively-oriented boundary of the region lying between the graphs of $y = x$ and $y = \sqrt{x}$. Evaluate your integral by hand.



5. (5 points) Find the critical point(s) and relative extreme value(s) of

$$f(x, y) = xy + 1/x + 8/y.$$

