Math 233 - Final Exam A

Name _____

December 9, 2022

Score _____

Show all work to receive full credit. Supply explanations where necessary. Place your final answer in the box provided. This test is due December 15. You must work individually.

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1. (5 points) Use a double integral to find the area of the 1st quadrant region inside both the circle $r = 2\cos\theta$ and the circle $r = 4\sin\theta$. After sketching the region and setting up the required iterated integral(s), you may use technology to evaluate the integral(s). Round your final answer to four decimal places.

2. (5 points) A solid lies in the 1st octant bounded by the coordinates planes, the plane $y + z = 2$, and the parabolic cylinder $x = 4 - y^2$. The density of the solid at the point (x, y, z) is given by $\rho(x, y, z) = 2 + z + \sin x$. Set up the triple integral that gives the mass of the solid. Use technology to evaluate the integral. Round your final answer to four decimal places.

3. (10 points) Let $\vec{F}(x, y) = (y^3 + 1)\hat{\imath} + (3xy^2 + 1)\hat{\jmath}$.

(a) Use our test to show that $\vec{F}$ is a conservative vector field. Then find a scalar potential function for $\vec{F}$. Write the potential function in the box below.

(b) Evaluate $\int_C \vec{F}(x, y) \cdot d\vec{r}$ where C is the semicircular path in the 1st quadrant from $(0, 0)$ to $(2, 0)$. (Hint: Use the fact that $\vec{F}$ is conservative.)

4. (5 points) Use Green's Theorem to evaluate

$$\int_C \cos y \, dx + (xy - x \sin y) \, dy,$$

where C is the positively-oriented boundary of the region lying between the graphs of $y = x$ and $y = \sqrt{x}$. Evaluate your integral by hand.

5. (5 points) Find the critical point(s) and relative extreme value(s) of

$$f(x, y) = xy + 1/x + 8/y.$$