Math 233 - Test 3

November 10, 2022

Name ______________________________

Score __________

Show all work to receive full credit. Supply explanations where necessary. You must work individually on this test. The test is due at 9:30 am on November 15.

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1. (6 points) Let $f(x,y) = \tan^{-1}(\frac{y}{x})$. Evaluate f_x and f_y at the point $(2,-2)$.

2. (4 points) Think about the graph of the function $f(x,y) = 4 - x^2 - y^2$. Without computing any derivatives, explain how/why we should know that $f_x(2,0)$ is negative.

3. (4 points) The function f is defined below. It is continuous everywhere, but it can be shown that $f_{xy}(0,0) \neq f_{yx}(0,0)$. Use our theorem about the equality of mixed partial derivatives to draw a conclusion about f_{xy} or f_{yx}.

$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

4. (6 points) For $x + y \neq 0$, let $g(x, y, z) = \dfrac{7xz}{x + y}$. Compute g_{xzyz}. Show work or explain your reasoning.

5. (8 points) Let $w = x^2yz^3 + \sin(yz)$. Use differentials to estimate the change in w that accompanies the change in (x, y, z) from the point $(2, \pi, 1)$ to $(1.9, 3, 1.1)$.

6. (10 points) The total resistance R of two resistors connected in parallel satisfies the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2},$$

where R_1 and R_2 are the resistances of the connected resistors. Find the linearization of R at $(R_1, R_2) = (10, 15)$ and use it to approximate the total resistance when $R_1 = 10.4$ and $R_2 = 14.7$.

7. (6 points) Suppose that z is a function of x and y. Further suppose that x and y are converted to polar coordinates by $x = r\cos\theta$ and $y = r\sin\theta$. Write the formulas for $\partial z/\partial r$ and $\partial z/\partial\theta$.

8. (6 points) Suppose that y is implicitly defined as a function of x and z by the equation

$$3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0.$$

Find $\partial y/\partial z$.

9. (6 points) Compute the directional derivative of $g(x, y, z) = xye^z$ at $(2, 4, 0)$ in the direction from $(2, 4, 0)$ to $(0, 0, 0)$.

10. (6 points) The temperature at the point (x, y) on a metal plate is given by

$$T = \frac{x}{x^2 + y^2}.$$

Find the direction of greatest increase in heat from the point $(3, 4)$.

11. (10 points) Find the point(s) on the surface $z = xy + \frac{1}{x} + \frac{1}{y}$ at which the tangent plane is horizontal.

12. (10 points) Find and classify the critical points of $f(x, y) = -x^3 + 4xy - 2y^2 + 1$.

13. (8 points) Find a set of parametric equations for the line normal to the graph of $y \ln(xz^2) = 2$ at the point $(e, 2, 1)$.

14. (10 points) Consider the iterated integral $\int_0^{\sqrt{2}/2} \int_y^{\sqrt{1-y^2}} xy\, dx\, dy$.

(a) Sketch the region of integration (in detail).

(b) Evaluate the iterated integral.