Math 233 - Test 2

October 13, 2022

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Find the function $\vec{r}(t)$ that satisfies

$$\vec{r}\,'(t) = e^{-2t}\,\hat{\imath} - t\,\hat{\jmath} + \frac{5t}{t^2+1}\,\hat{k}; \qquad \vec{r}(0) = 5\hat{\imath} + 3\hat{\jmath} - 2\hat{k}.$$

2. (10 points) Let $\vec{r}(t) = (t^2 - t)\,\hat{\imath} + \frac{1}{6}(4t-1)^{3/2}\,\hat{\jmath} + 5\,\hat{k}$. Starting from $t = 1$, reparameterize $\vec{r}$ in terms of the arc-length parameter s.

3. (8 points) For $-\pi/2 < x < \pi/2$, let $f(x) = \ln(\cos x)$. Compute the curvature function and say where the graph of f has its maximum curvature.

4. (2 points) An object is moving along a curve in such a way that the normal component of its acceleration is zero. What can you say about the motion of the object?

5. (2 points) An object is moving along a curve in such a way that the tangential component of its acceleration is zero. What can you say about the motion of the object?

6. (6 points) Consider the surface described by the equation $z - 4y^2 = x^2$.

   (a) Fix a value for one of the variables and draw a good sketch of the corresponding level curve.

   (b) Identify the surface.

   (c) Sketch a rough graph of the surface.

7. (6 points) Consider the function $F(x, y) = \sqrt{y^2 - 2x^2}$.

   (a) What is the domain of F?

   (b) Sketch the level curve $F(x, y) = 0$.

   (c) Describe the graph of F.

8. (20 points) A rock is thrown downward from the top of a building that is 168 ft high at an angle of 60° below the horizontal. The initial speed of the rock is 80 ft/sec. Ignore air resistance and use $g = 32\,\text{ft/sec}^2$.

   (a) When will the rock hit the ground?

   (b) How far from the base of the building will the rock land?

   (c) What will be the speed of the rock when it hits the ground?

   (d) Set up the definite integral that gives the length of the path of the rock. Use your calculator to estimate the value of your integral.

9. (6 points) Describe what it means to be a neighborhood of the point $(0, 0)$.

Follow-up problem: Let D be the set of all points in the circle $x^2 + y^2 = 1$ for which $x \neq y$. More formally,

$$D = \{(x, y) : x^2 + y^2 < 1 \text{ and } x \neq y\}.$$

Explain why D is NOT a neighborhood of $(0, 0)$.

10. (12 points) Use the two-path test to show that each limit fails to exist.

(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2 + y^2}$

(b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^4 + y^2}$

11. (10 points) Compute each limit.

(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 16y^2}{x^2 + 4y}$

(b) $\lim\limits_{(x,y)\to(2,2)} \dfrac{3x - 3y}{\sqrt{x} - \sqrt{y}}$

12. (8 points) Let $\vec{r}(t) = -\cos 3t\,\hat{\imath} - \sin 3t\,\hat{\jmath} + 4t\,\hat{k}$. Compute $\hat{N}(t)$.