Math 233 - Quiz 2 (IC)

September 1, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Find a unit vector that is orthogonal to $\vec{w} = 2\hat{\imath} + 8\hat{\jmath} - 7\hat{k}$.

Let $\vec{v} = -8\hat{\imath} + 2\hat{\jmath}$

$\vec{v} \cdot \vec{w} = -16 + 16 + 0 = 0$

$\Rightarrow$ $\vec{v}$ is orthog to $\vec{w}$.

$\|\vec{v}\| = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}$

$$\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{17}}\left(-4\hat{\imath} + \hat{\jmath}\right)$$

2. (2 points) Find the measure of the angle at vertex B in triangle ABC. Write your answer in degrees, rounded to the nearest tenth.

$A(1,1,3)$, $B(3,6,5)$, $C(-1,-2,5)$

$\vec{BA} = -2\hat{\imath} - 5\hat{\jmath} - 2\hat{k}$ $\quad \|\vec{BA}\| = \sqrt{33}$

$\vec{BC} = -4\hat{\imath} - 8\hat{\jmath} + 0\hat{k}$ $\quad \|\vec{BC}\| = \sqrt{80}$

$\vec{BA} \cdot \vec{BC} = 8 + 40 + 0 = 48$

$$\cos\theta = \frac{48}{\sqrt{33 \cdot 80}} \Rightarrow \theta \approx 20.9^\circ$$

3. (1 point) Find the projection of $\vec{y}$ onto $\vec{x}$, where $\vec{x} = -\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{y} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

$$\text{proj}_{\vec{x}}\,\vec{y} = \frac{\vec{x} \cdot \vec{y}}{\vec{x} \cdot \vec{x}}\,\vec{x} = \frac{-7}{11}\left(-\hat{\imath} + 3\hat{\jmath} - \hat{k}\right)$$

$$= \frac{7}{11}\hat{\imath} - \frac{21}{11}\hat{\jmath} + \frac{7}{11}\hat{k}$$

Math 233 - Quiz 2 (TH)

September 1, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due September 6.

1. (2 points) Find a unit vector that is orthogonal to both $\vec{x} = -5\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{y} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

$$\vec{z} = \vec{x} \times \vec{y} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -5 & 3 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 5\hat{\imath} + 7\hat{\jmath} - 4\hat{k}$$

$$\frac{\vec{z}}{\|\vec{z}\|} = \frac{1}{3\sqrt{10}}\left(5\hat{\imath} + 7\hat{\jmath} - 4\hat{k}\right)$$

$$\|\vec{z}\| = \sqrt{25+49+16} = \sqrt{90} = 3\sqrt{10}$$

2. (1 point) If $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$, must it be true that $\vec{v} = \vec{w}$?

No, for example, if $\vec{u} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{v} = 5\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$, and $\vec{w} = 8\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$, then $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w} = -13$. However, $\vec{v} \neq \vec{w}$.

What must be true is that $\vec{u} \cdot (\vec{v} - \vec{w}) = 0$.

3. (2 points) Find parametric and symmetric equations for the line through the points $P(5, 7, -3)$ and $Q(6, -2, 3)$.

$$\vec{PQ} = \hat{\imath} - 9\hat{\jmath} + 6\hat{k}$$

Using P and $\vec{v} = \vec{PQ}$...

Parametric

$$x = 5 + t$$
$$y = 7 - 9t$$
$$z = -3 + 6t$$

Symmetric

$$x - 5 = \frac{y-7}{-9} = \frac{z+3}{6}$$