Math 233 - Quiz 1 (IC)

August 25, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) In component form, $\vec{v} = \langle 8, -15 \rangle$. Find the magnitude of $-3\vec{v}$.

$$\|\vec{v}\| = \sqrt{64 + 225} = \sqrt{289} = 17$$

$$\|-3\vec{v}\| = 3\|\vec{v}\| = 3 \cdot 17 = \boxed{51}$$

2. (2 points) The vector $\vec{u}$ has initial point $P(5, -4)$ and terminal point $Q(-2, 3)$. The vector $\vec{w}$ has the same direction as $\vec{u}$ but has magnitude 4. Write $\vec{w}$ in component form.

$$\vec{u} = \overrightarrow{PQ} = \langle -7, 7 \rangle \qquad \|\vec{u}\| = 7\sqrt{2}$$

$$\frac{\vec{u}}{\|\vec{u}\|} = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$\vec{w} = \left\langle \frac{-4}{\sqrt{2}}, \frac{4}{\sqrt{2}} \right\rangle = \langle -2\sqrt{2}, 2\sqrt{2} \rangle$$

Math 233 - Quiz 1 (TH)

August 25, 2022

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. This quiz is due August 30.

1. (2 points) What is the slope of the vector $\vec{w} = 4\hat{\imath} - 3\hat{\jmath}$? Find a unit vector perpendicular to $\vec{w}$.

Slope...

$m_\perp = \frac{4}{3}$

$(0,0)$ & $(4,-3)$

$\vec{u}$ = Perp vector $= 3\hat{\imath} + 4\hat{\jmath}$ $\quad \|\vec{u}\| = 5$

$\Rightarrow$ $m = \frac{-3}{4}$

$\frac{\vec{u}}{\|\vec{u}\|} = \frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}$

2. (1 point) Write the component form of the 2D vector that has magnitide 6 and makes a 210° angle with the positive x-axis.

$\vec{v} = 6\cos 210^\circ\,\hat{\imath} + 6\sin 210^\circ\,\hat{\jmath}$

$= 6\left(-\frac{\sqrt{3}}{2}\right)\hat{\imath} + 6\left(-\frac{1}{2}\right)\hat{\jmath} = -3\sqrt{3}\,\hat{\imath} - 3\hat{\jmath}$

3. (1 point) Find the unit vector whose direction is the opposite of $\vec{v} = 4\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

$\|\vec{v}\| = \sqrt{16 + 1 + 4} = \sqrt{21}$

$\frac{-\vec{v}}{\|\vec{v}\|} = -\frac{4}{\sqrt{21}}\hat{\imath} - \frac{1}{\sqrt{21}}\hat{\jmath} + \frac{2}{\sqrt{21}}\hat{k}$

4. (2 points) Determine the angle between the vectors $\vec{x} = 6\hat{\imath} - 5\hat{k}$ and $\vec{y} = -\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$. Write your final answer in degrees, rounded to the nearest tenth.

$\cos\theta = \frac{\vec{x}\cdot\vec{y}}{\|\vec{x}\|\,\|\vec{y}\|} = \frac{-6 + 0 + 10}{\sqrt{61}\,\sqrt{14}} = \frac{4}{\sqrt{854}} \approx 0.136877$

$\theta \approx 82.1^\circ$