Math 233 - Test 3a

November 11, 2021

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Discuss the continuity of each function. Explain your reasoning.

   (a) $f(x, y) = \ln(x + y)$

   (b) $g(x, y, z) = e^{5xyz}$

2. (8 points) Let $z = \sin(x^2y - 2x + 4)$.

   (a) Compute $\partial z/\partial x$.

   (b) Compute $\partial z/\partial y$.

   (c) If you were on the graph of the equation at the point $(2, 0, 0)$ and you looked in the direction of the positive x-axis, would you be looking uphill or downhill? Explain.

   (d) If you were on the graph of the equation at the point $(2, 0, 0)$ and you looked in the direction of the positive y-axis, would you be looking uphill or downhill? Explain.

3. (9 points) Suppose $z = 2xe^{5y} - 3ye^{-x}$.

   (a) Which first partial derivative should be computed first in order to obtain $\dfrac{\partial^2 z}{\partial x \partial y}$?

   (b) Do you expect to have $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y \partial x}$? Explain.

   (c) Compute $\dfrac{\partial^2 z}{\partial x \partial y}$.

4. (8 points) Suppose $w = f(x, y)$, where $x = u - v$ and $y = v - u$. Use the chain rule to show that $\dfrac{\partial w}{\partial u} + \dfrac{\partial w}{\partial v} = 0$.

5. (6 points) Suppose z is implicitly defined as a function of x and y by the equation

$$x \ln y + y^2 z + z^2 = 8.$$

Find $\partial z/\partial x$ and $\partial z/\partial y$.

6. (6 points) The temperature at the point (x, y) on a metal plate is given by

$$T(x, y) = 4 + \sin(xy) + x + xy, \quad -2 \leq x \leq 2, \, -2 \leq y \leq 2.$$

(a) Find the direction of greatest increase in temperature from the point $(0, 1)$.

(b) At which point is there no increase or decrease in temperature regardless of which direction we look?

7. (5 points) Find an equation of the plane tangent to the surface $xy^2 + 3x - z^2 = 8$ at the point $(1, -3, 2)$.

8. (6 points) Find and classify the critical points of $f(x, y) = x^2 - 2xy + 2y^2 - 2x + 2y + 1$.

9. (6 points) Find the directional derivative of $f(x, y) = e^y \sin x$ at the point $(0, 0)$ in the direction toward the point $(2, 1)$.

Follow-up question: What does your directional derivative actually measure?