Math 233 - Test 3a

November 11, 2021

Name	
	Score

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Discuss the continuity of each function. Explain your reasoning.

(a)
$$f(x,y) = \ln(x+y)$$

(b)
$$g(x, y, z) = e^{5xyz}$$

- 2. (8 points) Let $z = \sin(x^2y 2x + 4)$.
 - (a) Compute $\partial z/\partial x$.
 - (b) Compute $\partial z/\partial y$.
 - (c) If you were on the graph of the equation at the point (2,0,0) and you looked in the direction of the positive x-axis, would you be looking uphill or downhill? Explain.
 - (d) If you were on the graph of the equation at the point (2,0,0) and you looked in the direction of the positive y-axis, would you be looking uphill or downhill? Explain.

- 3. (9 points) Suppose $z = 2xe^{5y} 3ye^{-x}$.
 - (a) Which first partial derivative should be computed first in order to obtain $\frac{\partial^2 z}{\partial x \partial y}$?
 - (b) Do you expect to have $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$? Explain.
 - (c) Compute $\frac{\partial^2 z}{\partial x \partial y}$.

4. (8 points) Suppose w = f(x, y), where x = u - v and y = v - u. Use the chain rule to show that $\frac{\partial w}{\partial u} + \frac{\partial w}{\partial v} = 0$.

5. (6 points) Suppose z is implicitly defined as a function of x and y by the equation

$$x \ln y + y^2 z + z^2 = 8.$$

Find $\partial z/\partial x$ and $\partial z/\partial y$.

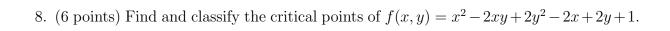
6. (6 points) The temperature at the point (x, y) on a metal plate is given by

$$T(x,y) = 4 + \sin(xy) + x + xy, \quad -2 \le x \le 2, \ -2 \le y \le 2.$$

(a) Find the direction of greatest increase in temperature from the point (0,1).

(b) At which point is there no increase or decrease in temperature regardless of which direction we look?

7. (5 points) Find an equation of the plane tangent to the surface $xy^2 + 3x - z^2 = 8$ at the point (1, -3, 2).



9. (6 points) Find the directional derivative of $f(x,y) = e^y \sin x$ at the point (0,0) in the direction toward the point (2,1).

Follow-up question: What does your directional derivative actually measure?