Math 216 - Final Exam A

May 7, 2014

Name ____________________

Score ____________________

Show all work. Supply explanations when necessary. You must work individually on this exam.

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1. (12 points) Solve the following system of differential equations.

$$x' = x + 3y - 2t^2$$

$$y' = 3x - y + t + 5$$

2. (8 points) Use Lapace transforms to solve: $y' - y = 1 + te^t,\ y(0) = 0$

3. (15 points) Use Laplace transforms to solve the system of equations.

$$x'' + 10x - 4y = 0$$

$$-4x + y'' + 4y = 0$$

$$x(0) = 0,\ x'(0) = 1,\ y(0) = 0,\ y'(0) = -1$$

Math 216 - Final Exam B

May 14, 2014

Name ____________________

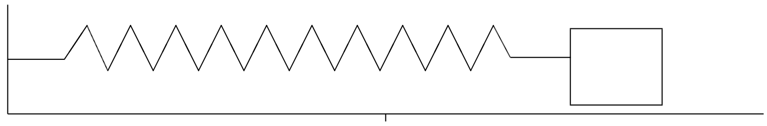
Score ______________

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) Solve: $y' = xy^2 \cos x, \quad y(\pi) = 2$

2. (10 points) Solve: $y'' + 2y' - 8y = 0, \quad y(0) = y'(0) = 6$

3. (20 points) A 2-kg mass is attached to a spring with spring constant 24 N/m. The damping constant for the system is 8 N-sec/m. The mass starts at the equilibrium position with an initial speed of 2 m/sec to the left. Is this mass-spring system underdamped, overdamped, or critically damped? Set up and solve the initial value problem that describes the displacement of the mass from equilibrium.



4. (10 points) Use Euler's method with a step size of $h = 0.5$ to approximate $y(3)$, where $y(x)$ is the solution of the initial value problem $y' = xy^2,\ y(2) = 1$.

5. (5 points) For $x > 0$, let $y_1(x) = \ln x^5$ and $y_2(x) = \ln x$. Compute the Wronskian of y_1 and y_2. Briefly explain why $y(x) = c_1y_1(x) + c_2y_2(x)$ cannot be the general solution of a 2nd-order, linear, homogeneous differential equation.

6. (12 points) According to Newton's Law of Cooling, the temperature T at time t of an object cooling in a medium of constant temperature M is described by the differential equation

$$\frac{dT}{dt} = k(M - T),$$

where k is some constant.

(a) Solve the differential equation.

(b) An object at 120°F is moved into a large room with an ambient temperature of 72°F. The object cools to 100°F in 6 min. Use your result from part (a) to find a formula for the temperature of the object at time t.

7. (20 points) Use variation of parameters to solve the following differential equation.

$$x'' - 2x' + x = \frac{e^t}{t^2}$$

8. (10 points) Solve: $(xy - 1)\,dx + (x^2 - xy)\,dy = 0$

9. (10 points) Find the orthogonal trajectories for the family of curves described by the equation $Cy^2 = x^3$.

10. (6 points) Given below are the differential equations or the equations of motion of some mass-spring systems. Each describes exactly one of the following situations: *simple harmonic motion*, *underdamped motion*, *overdamped motion*, or *critically damped motion*. Match each equation with the corresponding situation.

(a) $x(t) = 2e^{-2t} + 5te^{-2t}$

(b) $x'' + 8x' + 17x = 0$

(c) $x(t) = \sqrt{6}\sin(4t + \pi)$

(d) $2x'' + 5x' + 3x = 0$