Math 216 - Test 3

Name _____

April 30, 2014

Score _____

Show all work. Supply explanations when necessary. You must work individually on this exam.

1. (10 points) Solve: $y'' - 2y' + y = 0$; $y(0) = 5$, $y'(0) = 10$

2. (14 points) Solve: $y'' - 5y' + 4y = 8e^x + 5x + 2$

3. (14 points) Solve: $y'' + 3y' + 2y = \sin e^x$

4. (10 points) Solve the Cauchy-Euler equation:

$$4x^2y'' + 17y = 0; \ y(1) = -1, \ y'(1) = -\frac{1}{2}$$

5. (12 points) The equation $y''' - 6y'' + 11y' - 6y = 3x$ has a particular solution $y_p(x) = -\frac{11}{12} - \frac{1}{2}x$. Find the general solution of the equation

$$y''' - 6y'' + 11y' - 6y = 12x.$$

6. (13 points) A 1/2-kg mass is attached to a spring with spring constant $5\,\text{N/m}$. The damping constant for the system is $1\,\text{N-sec/m}$. The mass is moved $2\,\text{m}$ to the left of equilibrium (compressing the spring) and released from rest. Find the equation of motion. Write your final result in terms of a single trig function with phase shift. Graph your solution[1] and attach a copy.

$$x = 0$$
(Equilibrium)
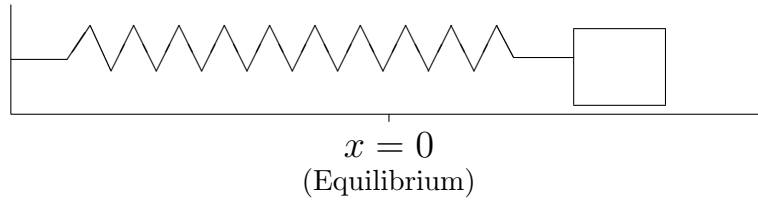
[1]If you don't have a good plotting program, try one that is available online such as http://fooplot.com.

7. (12 points) Consider the equation $x^2 y'' - 4xy' + 6y = 0$, $0 < x < \infty$. Do not solve this differential equation.

(a) Verify that $y_1(x) = x^3$ and $y_2(x) = x^2$ are solutions.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(0, \infty)$.

(c) Use what you've learned in parts (a) and (b) to find the solution of the IVP $x^2 y'' - 4xy' + 6y = 0$; $y(1) = 5$, $y'(1) = 16$.

(d) Is your solution in part (c) unique? Explain.

8. (15 points) A forced mass-spring system is governed by the IVP

$$0.2\frac{d^2x}{dt^2} + 1.2\frac{dx}{dt} + 2x = 5\cos 4t; \ x(0) = \frac{1}{2}, \ x'(0) = 0.$$

Find the transient and steady-state solutions. Identify which is which. Also sketch the graph of the equation of motion.