Math 216 - Test 2a

March 14, 2012

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) According to Newton's Law of Cooling, the temperature T at time t of an object cooling in a medium of constant temperature M is described by the differential equation

$$\frac{dT}{dt} = k(M - T),$$

where k is some constant.

(a) Solve the differential equation.

(b) An object at 120°F is moved into a large room with an ambient temperature of 72°F. The object cools to 100°F in 6 min. Use your result from part (a) to find a formula for the temperature of the object at time t.

(c) When will the object reach 76°F?

2. (10 points) Solve: $y'' - y' - 30y = 0;\ y(0) = 2,\ y'(0) = -2$

3. (6 points) Solve: $4y'' + 20y' + 25y = 0$

4. (12 points) A 50-gal tank initially contains 20 gal of pure water. A saltwater solution containing 0.5 lb of salt for each gallon of water begins entering the tank at a rate of 4 gal/min. Simultaneously, a drain is opened at the bottom of the tank, allowing the solution to leave the tank at a rate of 2 gal/min. What is the salt content (in pounds) in the tank at the precise moment that the tank is full?

5. (10 points) Consider the one-parameter family of curves described by

$$4x^2 + y^2 = C.$$

Find the family of orthogonal trajectories.

Math 216 - Test 2b Name ____________________
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1. (10 points) An object is launched upward so that its velocity (in m/s) at time t is described by the initial value problem

$$\frac{dv}{dt} = -19.6 - \frac{v}{20}, \quad v(0) = 200.$$

   (a) Solve the initial value problem.

   (b) If the initial height of the object is 30 m, find a formula for the height at time t.

   (c) Find the object's maximum height.

2. (10 points) Determine the recursive formulas for the Taylor method of order 3 for the IVP

$$\frac{dy}{dx} = xe^y, \quad y(0) = 1.$$

Then use $h = 0.1$ to approximate $y(0.2)$.

3. (10 points) Solve: $y^{(4)} - 3y'' - 4y = 0$; $y(0) = 1$, $y'(0) = 2$, $y''(0) = 3$, $y'''(0) = 4$

4. (5 points) Solve: $y'' - 2y' + 2y = 0$

5. (10 points) Consider the initial value problem

$$\frac{dy}{dx} = -20y, \quad y(0) = 1.$$

(a) Solve the IVP.

(b) Use the improved Euler's method with $h = 0.1$ to approximate $y(1)$. Be sure to look at your intermediate results.

(c) Compare your approximation with the exact value of $y(1)$.

(d) Use the improved Euler's method with $h = 0.01$ to approximate $y(1)$.

(e) Explain why you got the results you did in part (b).

6. (5 points) The solution of the initial value problem

$$\frac{dy}{dx} = y^2 - 2e^x y + e^{2x} + e^x, \quad y(0) = 3$$

has a vertical asymptote at a point in the interval $[0, 2]$. By experimenting with the classic 4th-order Runge-Kutta method, approximate this point.