Math 216 - Test 2a

October 27, 2010

Name ____________________

Score __________

Show all work. Supply explanations when necessary.

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1. (10 points) Solve: $y'' + 2y' - 8y = 0; \quad y(0) = 2,\ y'(0) = 10$

2. (12 points) Consider the equation $xy'' + 5y' = 0,\ 0 < x < \infty$.

(a) Verify that $y_1(x) = 1$ and $y_2(x) = \dfrac{1}{x^4}$ are solutions.

(b) Use the Wronskian to show that y_1 and y_2 are linearly independent on $(0, \infty)$.

(c) Use what you've learned in parts (a) and (b) to find the solution of the IVP $xy'' + 5y' = 0;\ y(1) = 2,\ y'(1) = 2$.

(d) Is your solution in part (c) unique? Explain.

3. (10 points) Use reduction of order to solve: $xy'' + y' = 4x$

4. (10 points) Solve: $(xy - 1)\,dx + (x^2 - xy)\,dy = 0$

5. (8 points) Solve: $y'' - 6y' + 9y = 0$

Math 216 - Test 2b

October 27, 2010

Name ______________________

Score __________

Show all work. Supply explanations when necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

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1. (12 points) Solve: $2yy'' = 1 + (y')^2; \quad y(0) = 1,\ y'(0) = 0$

2. (12 points)

(a) Find the recursive formula for the Taylor method of order three for the initial value problem $y' = e^{-y}, \quad y(0) = 0$.

(b) Use your method with $h = 0.1$ to approximate $y(0.3)$.

(c) Solve the IVP from part (a) and find the exact value of $y(0.3)$. What is the percent error in your approximation?

3. (12 points) Solve: $y'' + 10y' + 25y = 0; \quad y(0) = 2, \; y'(0) = 4$

4. (10 points) Consider the following initial value problem:

$$\frac{dy}{dx} = \frac{y-3}{x}, \quad y(1) = 5$$

(a) Use Euler's method with $h = 1$ to approximate $y(6)$. Do the computations by hand—they should be very easy.

(b) Solve the initial value problem and find the exact value of $y(6)$.

(c) Find the percent error in your approximation of $y(6)$? Why do you think Euler's method worked so well? Would it work as well with $h = 10$?

5. (4 points) An object is launched into the air so that its velocity, in meters per second, at any time t (in seconds) satisfies the initial value problem

$$v' = -0.8v - 9.8, \quad v(0) = 50.$$

Recalling that a launched object reaches a maximum height when its velocity is zero, use the fourth-order Runge-Kutta method to approximate when the object will reach its highest point. (If you're using the TI-83 program, the intermediate results are stored in your calculator's lists. To see the lists enter STAT 1.)