Math 216 - Quiz 2

September 8, 2010

Name key

Score _____

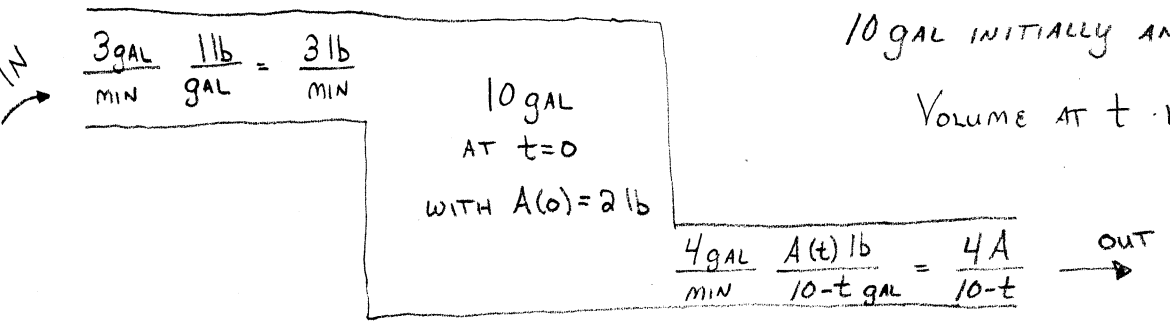
Show each step to receive full credit. Supply explanations when necessary.

1. (3 points) Problem #8, Page 16

Let A = amount of salt in lbs at time t

10 gal initially and losing 1 gal/min

Volume at t is $10-t$



$$\frac{dA}{dt} = 3 - \frac{4A}{10-t} \quad \text{or} \quad \frac{dA}{dt} + \frac{4}{10-t}A = 3$$

$$\mu(t) = e^{\int \frac{4}{10-t}dt} = e^{-4\ln|10-t|} = \frac{1}{(10-t)^4}$$

$$\frac{1}{(10-t)^4}A(t) = \int \frac{3}{(10-t)^4}dt = \frac{1}{(10-t)^3} + C$$

$$A(t) = (10-t) + C(10-t)^4$$

$$A(0) = 2 \Rightarrow 2 = 10 + C(10^4)$$

$$C = -\frac{8}{10^4}$$

$$\boxed{A(t) = (10-t) - \frac{8}{10^4}(10-t)^4}$$

2. (2 points) Solve the following Bernoulli equation:

$$x' = \frac{1}{t}x + \sqrt{x}$$

$$x' - \frac{1}{t}x = x^{1/2}$$

$$x^{-1/2}x' - \frac{1}{t}x^{1/2} = 1$$

$$u = x^{1/2}$$

$$\frac{du}{dt} = \frac{1}{2}x^{-1/2}\frac{dx}{dt}$$

$$2\frac{du}{dt} - \frac{1}{t}u = 1$$

$$\frac{du}{dt} - \frac{1}{2t}u = \frac{1}{2}$$

$$\mu(t) = e^{\int -\frac{1}{2t}dt} = e^{-\frac{1}{2}\ln|t|} = \frac{1}{\sqrt{t}}, \quad t > 0$$

$$\frac{1}{\sqrt{t}}u = \int \frac{1}{2\sqrt{t}}dt = \sqrt{t} + C$$

$$u = t + C\sqrt{t}$$

$$x^{1/2} = t + C\sqrt{t}$$

$$\boxed{x(t) = \left(t + C\sqrt{t}\right)^2 \quad \text{or} \quad x(t) = 0 \text{ IS ALSO A SOL'N.}}$$

3. (3 points) Solve: $\frac{\partial}{\partial y}\left(\frac{1}{x}+2y^2x\right) = 4yx = \frac{\partial}{\partial x}\left(2yx^2-\cos y\right)$

EQUATION IS EXACT.

$$\left(\frac{1}{x}+2y^2x\right)dx+(2yx^2-\cos y)\,dy=0, \quad y(1)=\pi$$

$$\frac{\partial F}{\partial x} = \frac{1}{x} + 2y^2x \Rightarrow F(x,y) = \ln|x| + x^2y^2 + g(y)$$

$$\frac{\partial F}{\partial y} = 2yx^2 - \cos y \Rightarrow F(x,y) = x^2y^2 - \sin y + h(x)$$

$$F(x,y) = x^2y^2 - \sin y + \ln|x|$$

$$x^2y^2 - \sin y + \ln|x| = C$$

$$y(1) = \pi \Rightarrow \pi^2 = C$$

$$\boxed{x^2y^2 - \sin y + \ln|x| = \pi^2}$$

4. (2 points) Solve:

$$\frac{dy}{dx}+\frac{3y}{x}+2=3x, \quad y(1)=1$$

STAND FORM

$$\frac{dy}{dx}+\frac{3}{x}y = 3x-2$$

$$\mu(x) = e^{\int \frac{3}{x}dx} = e^{3\ln|x|} = |x|^3 = x^3, \quad x>0$$

$$x^3y(x) = \int x^3(3x-2)\,dx = \frac{3}{5}x^5 - \frac{1}{2}x^4 + C$$

$$y(x) = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{C}{x^3}$$

$$y(1)=1 \Rightarrow \frac{3}{5}-\frac{1}{2}+C=1 \Rightarrow C = 1-\frac{6}{10}+\frac{5}{10} = \frac{9}{10}$$

$$\boxed{y(x) = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{9}{10x^3}, \quad x>0}$$