Math 201 - Program #2

February 24, 2011

Write, compile, and test a C++ program that implements Newton's method to solve the equation $f(x) = 0$, where f is a differentiable function of x. All floating-point numbers and functions should be declared `double`. You must use a main function and two other functions that define f and f', respectively.

Your program should approximate either the absolute error or the relative error in each iterate, depending on which is more appropriate. A symbolic constant should be used to hold the value of a floating-point number that will represent a "relative error threshold." An iterate whose absolute value is greater than this threshold will have its relative error approximated. Otherwise its absolute error will be approximated.

Two other symbolic constants should be defined: one to hold an integer value representing the maximum allowable number of iterations and one to hold a floating-point number representing the error tolerance. Your program should stop when the error tolerance has been met or the maximum number of iterations has been exceeded.

At each successful step of Newton's method, your program should output the iteration count, the value of the iterate, the approximate error in the iterate, and a character indicating whether the error approximation is absolute or relative. You need not output your starting guess (i.e. the zeroth iterate).

Be sure to check for division by zero!

1. Test your program by solving $10x^2 + 2x - 3 = 0$ starting with $x_0 = 1.0$, $x_0 = 5.0$, $x_0 = 100.0$, and $x_0 = 500.0$.
2. Test your program by solving $\cos(x) - x = 0$ starting with $x_0 = 1.0$, $x_0 = 5.0$, $x_0 = 100.0$, and $x_0 = 500.0$. Explain why Newton's method fails in certain cases.
3. Consider the equation $\cos(x) + \sin^2(50x) = 0$. One of the solutions is $x = \pi/2$. Try to determine how close to $\pi/2$ you must start in order for Newton's method to converge to this solution.
4. Use your program to approximate the only positive solution of

$$0.034525x^4 - 14.796277x^3 - 89.640787x^2 - 164.830547x - 90.020562 = 0.$$