

HW #1 Key

Problem 1:

a) Using 5-DIGIT ARITHMETIC,

$$f(a) = \sqrt{a+2} - \sqrt{2} = \sqrt{2} - \sqrt{2} = 0$$

RELATIVE ERROR IS

$$\frac{3.9322 \times 10^{-6} - 0}{3.9322 \times 10^{-6}} = \boxed{1 = 100\%}$$

$$b) f(x) = \frac{\sqrt{x+2} - \sqrt{2}}{1} \cdot \frac{\sqrt{x+2} + \sqrt{2}}{\sqrt{x+2} + \sqrt{2}} = \boxed{\frac{x}{\sqrt{x+2} + \sqrt{2}}}$$

c) Using 5-DIGIT ARITHMETIC,

$$\text{Improves } f(a) = \frac{1.1122 \times 10^{-5}}{2\sqrt{2}} = \frac{1.1122 \times 10^{-5}}{2.8284} \\ = \boxed{3.9323 \times 10^{-6}}$$

RELATIVE ERROR IS

$$\left| \frac{3.9322 \times 10^{-6} - 3.9323 \times 10^{-6}}{3.9322 \times 10^{-6}} \right| = \boxed{2.5431 \times 10^{-5}}$$

Problem 2:

$$a) \quad p(x) = x^4 + 5x^3 + 8x^2 - 7x + 2$$

$$= x \cdot x \cdot x \cdot x + 5 \cdot x \cdot x \cdot x + 8 \cdot x \cdot x - 7 \cdot x + 2$$

13 operations required to evaluate $p(x)$

$$p(2) = 16 + 40 + 32 - 14 + 2 \\ = \underline{\underline{76}}$$

b) NESTED FORM

$$p(x) = (((x+5)x+8)x-7)x+2$$

c)

7 operations required to evaluate $p(x)$

$$p(2) = \underline{\underline{76}}$$

Problem 3: $x^2 - 26x + 1 = 0$

a) EXACT SOLUTIONS: $x = \frac{26 \pm \sqrt{26^2 - 4}}{2}$

$$x_1 = \frac{26 + \sqrt{672}}{2} \approx 25.9614814$$

$$x_2 = \frac{26 - \sqrt{672}}{2} \approx 0.0385186032$$

} EACH WITH
9 sig.
DIGITS.

b) Using 5-DIGIT ARITHMETIC WITH ROUNDING...

$$x_1 = \frac{26 + \sqrt{672}}{2} = \frac{26 + 25.923}{2} = \frac{51.923}{2} = \boxed{25.962}$$

$$\text{REL ERROR IN } x_1 \text{ IS } \boxed{1.9976 \times 10^{-5}}$$

$$x_2 = \frac{26 - \sqrt{672}}{2} = \frac{26 - 25.923}{2} = \frac{0.077}{2} = \boxed{0.038500}$$

$$\text{REL ERROR IN } x_2 \text{ IS } \boxed{4.8297 \times 10^{-4}}$$

NOTICE THAT THE ERROR IN x_2 IS

★ APPROXIMATELY 24 TIMES THE ERROR
IN x_1 .

c) Improved QF using 5-digit ARITHMETIC...

$$q = -0.5 \left(-26 - \sqrt{672} \right) = -0.5 (-51.923)$$

$$x_1 = q/a, \quad x_2 = c/q = 25.962$$

$$x_1 = \boxed{25.962}$$

$$\text{REL error in } x_1 \text{ is } \boxed{1.9976 \times 10^{-5}}$$

$$x_2 = \frac{1}{25.962} = \boxed{0.038518}$$

$$\text{REL error in } x_2 \text{ is } \boxed{1.5660 \times 10^{-5}}$$

★ NOTICE THAT THE RELATIVE ERRORS
ARE ALMOST THE SAME.