Math 200 - 1st Final Exam

May 4, 2011

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. Multiple choice problems are worth 0, 1, or 2 points depending on your answer and the work shown.

1. Choose the division model that best fits the following problem situation: *Sammy's mother made 36 cookies for the school bake sale. She would like to put them into bags that hold 4 cookies each. How many bags does she need?*

   (a) partition

   (b) missing factor

   (c) repeated subtraction

   (d) Cartesian product

2. Which one of the following facts illustrates the commutative property of multiplication?

   (a) $(2 + x) + y = y + (2 + x)$

   (b) $3 \cdot (x + 7) + 2 = 3x + 23$

   (c) $3(2)(x + y) = 2(3)(x + y)$

   (d) $(2x + 3y) + 8 = 2x + (3y + 8)$

3. What is the value of the digit 5 in the base-eight numeral 6543_{eight}?

   (a) 5

   (b) $5 \cdot 64$

   (c) 500

   (d) $5 \cdot 512$

4. Suppose $A = \{x, y, z\}$, $B \subseteq A$, and $n(B) = 3$. Which one of the following must be true?

   (a) $(x, x) \in A \times B$

   (b) $A \cap B = \emptyset$

   (c) $n(A \cup B) = 6$

   (d) $B - A = 3 - x - y - z$

5. If m and n are both negative integers, then what can be said about $-m - n$?

   (a) $-m - n$ is a positive integer.

   (b) $-m - n$ is a negative integer.

   (c) $-m - n$ is not an integer.

   (d) There is not enough information to say anything about $-m - n$.

6. Choose the multiplication model that best fits the following problem situation: *The graduates walked into the auditorium as a group in ten rows of four. How many graduates were there?*

   (a) Cartesian product
   (b) repeated addition
   (c) area/array
   (d) set partition

7. Convert the base-ten numeral 372 to base-four.

   (a) 11310_{four}
   (b) 78_{four}
   (c) 1131_{four}
   (d) 1032_{four}

8. When using the 4-step, problem-solving process which one of these strategies would NOT be considered part of understanding the problem?

   (a) Reread the problem.
   (b) State the problem in your own words.
   (c) Determine what information is not needed.
   (d) Write an equation.

9. Suppose $A = 2^3 \cdot 5^2 \cdot 7 \cdot 13^3$ and $B = 2 \cdot 3 \cdot 5^3 \cdot 13^2$. Find the LCM of A and B.

   (a) $2 \cdot 3 \cdot 5^2 \cdot 7 \cdot 13^2$
   (b) $2^3 \cdot 3 \cdot 5^3 \cdot 7 \cdot 13^3$
   (c) $2 \cdot 5^2 \cdot 13^2$
   (d) $2^4 \cdot 3 \cdot 5^5 \cdot 7 \cdot 13^5$

10. Which one of these numbers is the 1371st term of the following arithmetic sequence?

$$18, 25, 32, 39, 46, 53, 60, 67, \ldots$$

   (a) 9608
   (b) 9615
   (c) 9601
   (d) 9597

11. Which one of the following is an example of inductive reasoning?

    (a) $2(3+5) = 2(5+3)$

    (b) A sequence begins with 2,4,6,8. The next term must be 10.

    (c) If $x = 10$, then $2x + 3 = 23$.

    (d) Wednesdays are pizza days, so today is a pizza day.

12. Let W be the set of all whole numbers. The set A is defined below using set-builder notation. Which one of the given sets is equal to A?

$$A = \{x \mid x = 2n \text{ where } n \in W \text{ and } n < 2\}$$

    (a) $\{0, 2\}$

    (b) $\{0, 1\}$

    (c) $\{0, 2, 4, 6, \dots\}$

    (d) $\{\dots, -6, -4, -2, 0, 2\}$

13. Which one of the following is a valid test for divisibility by 20?

    (a) An integer is divisible by 20 if and only if its last digit is 0.

    (b) An integer is divisible by 20 if and only if the sum of its digits is divisible by 20.

    (c) An integer is divisible by 20 if and only if it is divisible by both 2 and 10.

    (d) An integer is divisible by 20 if and only if it is divisible by both 4 and 5.

14. (5 points) Use any multiplication algorithm to compute $413_{\text{five}} \times 42_{\text{five}}$.

15. (5 points) Test the number 749968830 for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, and 20. Show work and/or explain your reasoning.

16. (5 points) The area of a rectangle is 24 square inches. Its length and width are natural numbers. Use this information to find the rectangle with the least possible perimeter.

Width, w

Area $= \ell \cdot w = 24$

Perimeter $= 2\ell + 2w$

Length, ℓ
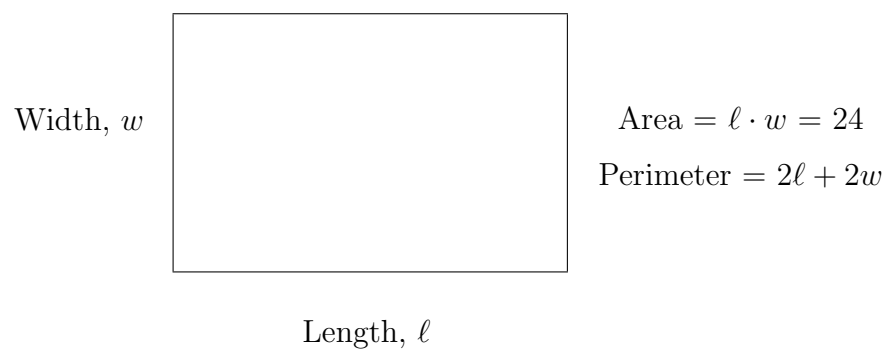
17. (5 points)

   (a) Find the fourth term of the geometric sequence whose first term is 3 and whose ratio is 5.

   (b) A recursive sequence is defined as follows:

   $$B_1 = -2, \quad B_n = -3 \cdot B_{n-1} + 5, \text{ for } n = 2, 3, 4, \ldots$$

   Find the third term of the sequence.

   (c) Find the next term: $2, 4, 9, 17, 28, 42, \ldots$

18. (5 points) Suppose A and B are subsets of U, and U has 36 elements. Use a two-set Venn diagram to help you determine $n(B)$ if $n(A) = 16$, $n(A \cap B) = 9$, and $n(\overline{A \cup B}) = 8$.

19. (2 points) Find the greatest common divisor of 60 and 168.

20. (3 points) Use a model to illustrate and compute each product. Use a different model for each part. (Model what is given, not a related problem.)

   (a) -3×4

   (b) $-3 \times (-2)$

21. (2 points) After looking at these examples:

$$5 \cdot 3 + 5 \cdot 5 = 40, \quad 5 \cdot 2 + 5 \cdot 4 = 30, \quad 5 \cdot 8 + 5 \cdot 10 = 90,$$

   Marcus conjectured that the sum of two multiples of 5 is a multiple of 10. Is he correct? If not, give a counterexample.

22. (5 points) Find the prime factorization of 4500. Then use your factorization to determine the number of positive integer divisors of 4500.

23. (5 points) Use a NONSTANDARD algorithm to compute each of the following.

   (a) $5686 + 6679$

   (b) $345 - 269$

   (c) $6745 \div 5$

24. (2 points) State a basic property of the Hindu-Arabic numeration system.

25. (1.5 points) Write an application problem (i.e. a word problem) involving subtraction in which the subtraction is best modeled using comparison.

26. (1.5 points) Write the expanded form of the number 54967.

27. (2 points) Use an integer subtraction model to illustrate and compute $-2 - (-5)$.