Math 200 - Test 1

September 19, 2012

Name _____

Score _____

SHOW ALL WORK TO RECEIVE FULL CREDIT. SUPPLY EXPLANATIONS WHERE NECESSARY.

1. (2 points) Clearly state the four steps of the problem-solving process (in order).

2. (3 points) Which step of the problem-solving process is being used in each of these situations?

   (a) While solving a complicated equation, Annie double checked her work at each step.

   (b) After completely solving a problem, John solved the problem again by using an entirely different approach.

   (c) Stacy wrote, "Let x be the number of childrens' tickets."

   (d) Fred thought to himself, "My solution cannot be correct because the length of a rectangle cannot be $-12\,\text{in}$."

   (e) A problem involved finding the perimeter of a rectangle. Sally decided to first find the length of the rectangle.

   (f) In order to solve a problem involving a trapezoid, Sam had to look up the definition of *trapezoid*.

3. (3 points) Sabrina bought two books at the book fair. One book cost $3 more than the other, and because she bought two books, she got a $2 discount. If she spent a total of $12, how much was each book?

4. (3 points) Tony builds regular bicycles and tandem (2-person) bicycles. Looking at his bicycles, he sees 18 wheels and 12 seats. How many of each type of bike does he have?

5. (4 points) Choose either one of the two problems above. For each step of the problem-solving process, list one strategy that you actually used in your solution. (Only list strategies that you actually used!)

6. (1 point) Which one of the following is an example of inductive reasoning?

   (a) $2(3+5) = 2(5+3)$

   (b) A sequence begins with 2,4,6,8. The next term must be 10.

   (c) If $x = 10$, then $2x + 3 = 23$.

   (d) Wednesdays are pizza days, so today is a pizza day.

7. (1 point) Which one of these sets is well-defined?

   (a) The set of all tall women

   (b) $\{1, 2, 3, 4, 5\}$

   (c) $\{x \mid x \in \mathbb{N} \text{ and } x \text{ is big}\}$

   (d) The set of all happy children

8. (1 point) Which one of these is a geometric sequence?

   (a) $1, 2, 3, 4, 5, \ldots$

   (b) $6, 7, 12, 21, 44, \ldots$

   (c) $1, 12, 123, 1234, \ldots$

   (d) $1, 2, 4, 8, 16, \ldots$

9. (1 point) Which one of the following is an example of deductive reasoning?

   (a) Every elephant I have ever seen is gray. Therefore every elephant must be gray.

   (b) It has rained every day. Therefore it will rain tomorrow.

   (c) Since the order of multiplication doesn't matter, $15 \cdot 37$ must be equal to $37 \cdot 15$.

   (d) A sequence begins with 1,2,3,4. The next term must be 5.

10. (1 point) Find the 87th term of the following arithmetic sequence: $10, 13, 16, 19, \ldots$

    (a) 87

    (b) 268

    (c) 271

    (d) 254

11. (3 points) The 1st difference of a sequence is $2, 4, 6, 8, 10 \ldots$. Find the first six terms of the original sequence if the sum of the first two terms (of the original) is 10.

12. (3 points) A scientist notices that her bacteria culture doubles in population every hour. If she started an experiment with 100 bacteria, how many will she have after n hours? What type of sequence is being described in this problem?

13. (3 points) Translate into words and then rewrite in roster notation.

$$\{x \mid x \in \mathbb{N} \text{ and } x \leq 8.9\}$$

14. (2 points) Rewrite in set-builder notation: $\{5, 6, 7, 8, 9\}$

15. (3 points) Find a formula for the nth term of each sequence.

    (a) $1, 8, 15, 22, 29, \ldots$

    (b) $17, 21, 25, 29, 33, \ldots$

    (c) $2, 10, 50, 250, 1250, \ldots$

16. (2 points) Find the next two terms: $6, 7, 12, 21, 34, \ldots$

17. (2 points) Give a counterexample: For any number x, $(x+1)^2 = x^2 + 1$.

18. (2 points) Let X be the set of letters of the word *racecar*.

    (a) Write X is roster notation.

    (b) Find $n(X)$.

19. (3 points) Suppose $A = \{1, 3, 7\}$, $B = \{x, y, z\}$ and $C = \{3, 6, 9, 12\}$. Indicate whether each statement is true or false.

    (a) $n(A) = 7$ _____

    (b) $3 \in A$ _____

    (c) $A \sim C$ _____

    (d) B is well-defined. _____

    (e) $A = B$ _____

    (f) A and C are in 1–1 correspondence. _____

20. (4 points) Consider the sum: $3 + 8 + 13 + \cdots + 2083$

    (a) How many terms does the sum have?

    (b) Compute the sum.

21. (1 point) Kate asked George to write the elements of the set $\{5, 10, 15\}$. George wrote: $\{5\}$, $\{10\}$, $\{15\}$. Is George correct? Explain.

22. (2 points) Find the cardinality of the set $\{2, 5, 8, \ldots, 320\}$.