Math 200 - Test 1

Name _____

September 22, 2010

Score _____

Show all work to receive full credit (even on multiple-choice problems). Supply explanations when necessary.

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1. (4 points) Clearly state the 4 steps of the problem-solving process (in order).

2. (1 point) When using the 4-step, problem-solving process which one of these strategies WOULD BE considered part of understanding the problem?

   (a) Look for a pattern.

   (b) Keep an accurate record of your work.

   (c) Construct a table or draw a picture.

   (d) Determine what information is unknown.

3. (3 points) State three different strategies for carrying out the plan.

4. (1 point) When using the 4-step, problem-solving process which one of these strategies WOULD NOT BE considered part of devising a plan?

   (a) Look for a pattern.

   (b) Determine what is known and unknown.

   (c) Work backward.

   (d) Guess and check.

5. (1 point) Jerry defined a sequence as follows:

> The first term is the number 8. To get any subsequent term, multiply the previous term by 4 and then subtract 5.

What type of sequence did Jerry define?

(a) An arithmetic sequence

(b) A geometric sequence

(c) A sequence with a fixed higher-order difference

(d) A recursive sequence

6. (1 point) Compute the following sum:

$$1 + 2 + 3 + 4 + \cdots + 136 + 137$$

(a) 9453

(b) $18,906$

(c) 9316

(d) 9248

7. (1 point) Which one of the following is NOT true about the empty set?

(a) Its cardinality is zero.

(b) It is a subset of every set.

(c) It can be denoted by $\{\emptyset\}$.

(d) It is equivalent to itself.

8. (1 point) Let $A = \{1, 2\}$. How many subsets does A have?

(a) 4

(b) 3

(c) 2

(d) 1

9. (1 point) Which one of the following sets is well defined?

(a) $\{x \mid x \text{ is a big number}\}$

(b) The set of all nice days

(c) $\{1, 7, f, 9, q, \frac{2}{3}\}$

(d) $\{p \mid p \text{ is a pretty flower}\}$

10. (3 points) Rewrite each of the following statements using mathematical symbols.

    (a) The empty set is a subset of the set A.

    (b) 6 is an element of the set H

    (c) The sets P and Q are equivalent.

11. (3 points) The following sequence could be either arithmetic or geometric depending on the numbers replacing the blanks. Find the first five terms of both possible sequences.

$$2, \_\_\_\_, 18, \_\_\_\_, \_\_\_\_, \ldots$$

12. (2 points) Kevin made the following conjecture:

> If a fixed number is added to each term of a geometric sequence, the new sequence will also be a geometric sequence.

    Give a counterexample to disprove the conjecture.

13. (1 point) Who is given credit for devising the technique we use for finding the sum of the terms of an arithmetic sequence?

    (a) Leonhard Euler

    (b) Kurt Gödel

    (c) Gerolamo Cardano

    (d) Carl Gauss

14. (4 points) Problem-solving problem: In our class, there are six times as many women as men. Using variables, write a formula relating the number of women to the number of men. (It should be clear how you have used the problem-solving process.)

15. (2 points) Sally made the following conjecture:

    The sum of two even numbers is an even number.

    In order to disprove the conjecture, Jonathon gave the counterexample $3 + 5 = 8$. What is wrong with Jonathon's reasoning?

16. (3 points) Which one of these numbers is the 691st term of the following arithmetic sequence?

$$18, 25, 32, 39, 46, \ldots$$

    (a) 4914
    (b) 4848
    (c) 4826
    (d) 7594

17. (2 points) A sequence is defined recursively as follows:

$$B_1 = 5; \quad B_n = 2 \cdot B_{n-1} - 3, \text{ for } n = 2, 3, 4, \ldots$$

    Find B_3 (the third term of the sequence).

    (a) 11
    (b) 7
    (c) 5
    (d) 27

18. (6 points) Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5\}$, and $C = \{4, 5, 6\}$. In each of the following, insert an appropriate symbol $\sim$, $\in$, or $\subseteq$ to make the statement true.

    (a) 2_____A

    (b) B_____A

    (c) C_____B

    (d) $\emptyset$_____C

    (e) A_____A

    (f) $\{5\}$_____C

19. (1 point) For which one of these sets is it true that $n(A) = 1$?

    (a) $A = \emptyset$
    (b) $A = \{0, 1\}$
    (c) $A = \{\emptyset\}$
    (d) $A = \{1, 11, 111, 1111, \dots\}$

20. (2 points) There are 511 terms in the following sequence. Find the sum of the terms.

$$1, 4, 7, 10, \dots, 1528, 1531$$

    (a) $261,632$
    (b) $391,170.5$
    (c) $1,172,746$
    (d) $391,426$

21. (3 points) Consider the following sequence:

$$1, 3, 8, 16, 27, 41, \ldots$$

(a) Is this sequence arithmetic? How do you know?

(b) Is this sequence geometric? How do you know?

(c) Find 3 terms that continue a possible pattern.

22. (2 points) What does it mean for two sets to be equivalent?

23. (1 point) Suppose U is the set of all PSC math students and M is the set of all Math 200 students. Describe an element of $\overline{M}$.

24. (1 point) Let W be the set of all whole numbers greater than 101. Write W using set-builder notation.