Math 173 - Quiz 4
February 19, 2015

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) A mortar shell is fired with a muzzle speed of 500 ft/sec. Find the angle of elevation of the mortar if the shell strikes a target located 1200 ft away. What is the maximum height of the shell?

$$\vec{r}(t) = 500\cos\theta\, t\,\hat{i} + \left(-16t^2 + 500\sin\theta\, t\right)\hat{j}$$

$$500\cos\theta\, t = 1200$$

$$-16t^2 + 500\sin\theta\, t = 0 \Rightarrow t\left(500\sin\theta - 16t\right) = 0$$

$$t = \frac{500\sin\theta}{16}$$

$$\frac{500\cos\theta\, 500\sin\theta}{16} = 1200$$

$$2\cos\theta\sin\theta = 0.1536$$

$$\sin 2\theta = 0.1536$$

$$\Rightarrow \theta \approx 0.077 \approx \boxed{4.4^\circ}$$

COULD ALSO USE 85.6°. THIS WOULD GIVE A FAR GREATER HEIGHT.

MAX HEIGHT WHEN

$$-32t + 500\sin\theta = 0$$

$$t = \frac{500\sin\theta}{32}$$

$$\approx 1.2036$$

WITH THIS t,

$$-16t^2 + 500\sin\theta\, t \approx \boxed{23.18 \text{ FT}}$$

2. (5 points) Let $\vec{r}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath}$. Compute $\frac{d}{dt}(\hat{T}(t) \times \hat{N}(t))$.

$$\vec{r}\,'(t) = -\sin t\,\hat{i} + \cos t\,\hat{j} \qquad \|\vec{r}\,'(t)\| = 1$$

$$\hat{T}(t) = -\sin t\,\hat{i} + \cos t\,\hat{j}$$

$$\hat{T}\,'(t) = -\cos t\,\hat{i} - \sin t\,\hat{j} \qquad \|\hat{T}\,'(t)\| = 1$$

$$\hat{N}(t) = -\cos t\,\hat{i} - \sin t\,\hat{j}$$

$$\hat{T}(t) \times \hat{N}(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin t & \cos t & 0 \\ -\cos t & -\sin t & 0 \end{vmatrix}$$

$$= \hat{k} \Rightarrow \frac{d}{dt}\left(\hat{T} \times \hat{N}\right) = \frac{d}{dt}\left(\hat{k}\right) = \vec{0}$$