Math 173 - Quiz 8

April 17, 2014

Name Key

Score ____________

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the absolute extreme values of $f(x, y) = y^2 - 4x$ subject to $x^2 + y^2 = 9$.

$f(x,y) = y^2 - 4x$

$g(x,y) = x^2 + y^2$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$x^2 + y^2 = 9$

$\Downarrow$

$-4 = 2\lambda x$

$2y = 2\lambda y \longrightarrow 2y - 2\lambda y = 0$

$x^2 + y^2 = 9 \qquad y = 0$ or $\lambda = 1$

$y = 0 \Downarrow x = \pm 3$

$\lambda = 1 \Downarrow x = -2 \Downarrow y = \pm\sqrt{5}$

Critical pts are $(3,0)$, $(-3,0)$, $(-2,\sqrt{5})$, $(-2,-\sqrt{5})$

$f(-3,0) = 12$

$\boxed{\begin{array}{l} f(3,0) = -12 \leftarrow \text{Constrained min} \\ f(-2,\pm\sqrt{5}) = 13 \leftarrow \text{Constrained max} \end{array}}$

2. (3 points) Find the absolute extreme values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to $z = 1 + 2xy$.

$f(x,y,z) = x^2 + y^2 + z^2$

$g(x,y,z) = z - 2xy$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$z = 1 + 2xy$

$\Downarrow$

$\left.\begin{array}{l} 2x = -2\lambda y \\ 2y = -2\lambda x \end{array}\right\} \Rightarrow 2x^2 = 2y^2 \Rightarrow x = \pm y$

$2z = \lambda$

$z = 1 + 2xy$

$x = y$

$2x = -2\lambda x$

$x = 0$ or $\lambda = -1$

$x = 0 \downarrow y = 0 \downarrow z = 1 \downarrow (0,0,1)$

$\lambda = -1 \downarrow z = -\frac{1}{2}$, $-\frac{1}{2} = 1 + 2x^2$ Impossible

$x = -y$

$2x = 2\lambda x$

$x = 0$ or $\lambda = 1$

$x = 0 \downarrow (0,0,1)$

$\lambda = 1 \downarrow z = \frac{1}{2}$, $\frac{1}{2} = 1 - 2x^2 \Downarrow x = \pm\frac{1}{2} \Downarrow y = \mp\frac{1}{2} \downarrow \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

$\boxed{\begin{array}{l} f(0,0,1) = 1 \leftarrow \text{Constrained min} \\ f\left(\pm\frac{1}{2}, \mp\frac{1}{2}, \frac{1}{2}\right) = \frac{3}{4} \leftarrow \text{Constrained max} \end{array}}$

3. (4 points) Find the points on the plane $x + 2y + z = 2$ closest to $(2, 0, 4)$.

MINIMIZE $f(x,y,z) = (x-2)^2 + y^2 + (z-4)^2$

$$\text{s.t. } g(x,y,z) = x + 2y + z = 2$$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$x + 2y + z = 2$

$\Downarrow$

$$2(x-2) = \lambda \Rightarrow 2x = \lambda + 4$$

$$2y = 2\lambda \Rightarrow 2y = 2\lambda$$

$$2(z-4) = \lambda \Rightarrow 2z = \lambda + 8$$

$$x + 2y + z = 2 \longrightarrow 2x + 4y + 2z = 4$$

$$\lambda + 4 + 4\lambda + \lambda + 8 = 4$$

$$6\lambda + 12 = 4$$

$$\lambda = -\frac{4}{3}$$

$\Downarrow$

$$\boxed{x = \frac{4}{3},\quad y = -\frac{4}{3},\quad z = \frac{10}{3}} \leftarrow$$

THESE MUST GIVE A MIN. THERE IS CLEARLY NO MAX.

MIN DISTANCE IS $\sqrt{\left(\frac{4}{3}-2\right)^2 + \left(-\frac{4}{3}\right)^2 + \left(\frac{10}{3}-4\right)^2}$

$$= \frac{4}{\sqrt{6}}$$