Math 173 - Quiz 8

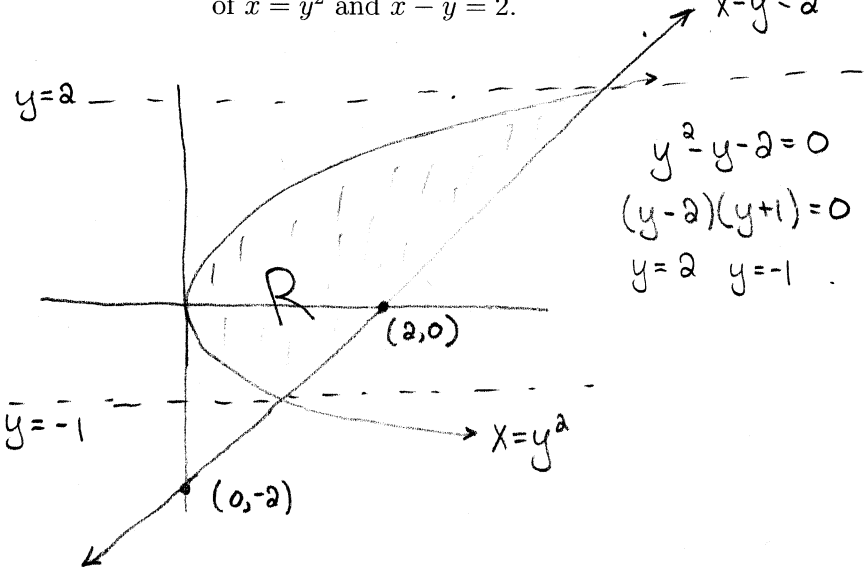
April 18, 2013

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2.5 points) Evaluate $\iint_R (2x - y)\, dA$, where R is the plane region between the graphs of $x = y^2$ and $x - y = 2$.



$y^2 - y - 2 = 0$

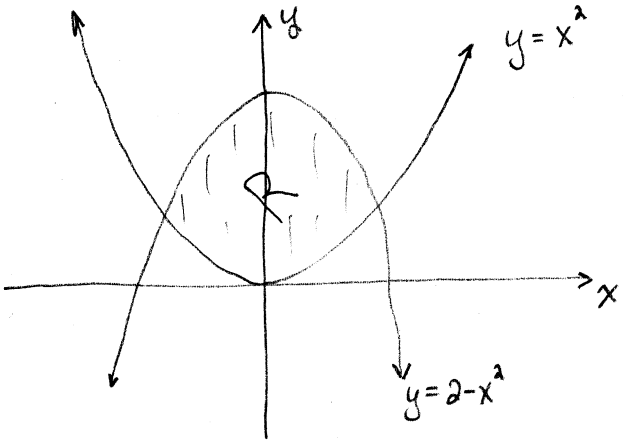
$(y-2)(y+1) = 0$

$y = 2 \quad y = -1$

$$\int_{-1}^{2} \int_{y^2}^{2+y} (2x - y)\, dx\, dy$$

$$= \frac{243}{40}$$

2. (2.5 points) Use a double integral to find the volume of the space region under the graph of $z = x^2 + y$ above the region in the xy-plane bounded by the graphs of $y = x^2$ and $y = 2 - x^2$.



$x^2 = 2 - x^2$

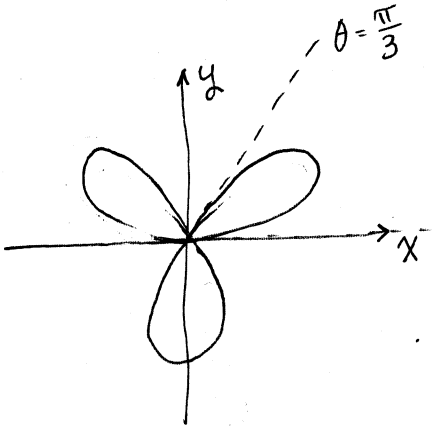
$x^2 = 1$

$x = \pm 1$

$$\int_{x=-1}^{x=1} \int_{y=x^2}^{y=2-x^2} (x^2 + y)\, dy\, dx$$

$$= \frac{16}{5}$$

3. (2.5 points) Use a double integral to find the area enclosed by one loop of the three-leaved rose $r = \sin 3\theta$.



$\sin 3\theta = 0$

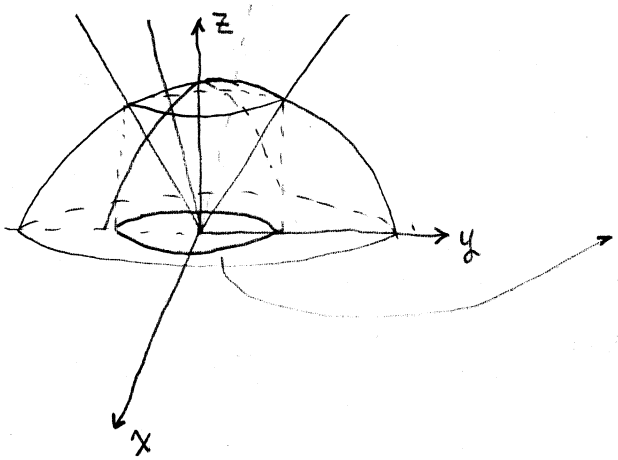
$\Rightarrow \theta = 0, \frac{\pi}{3}$

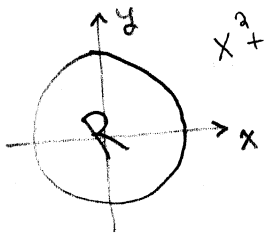
$$\text{Area} = \int_{\theta=0}^{\theta=\frac{\pi}{3}} \int_{r=0}^{r=\sin 3\theta} r \, dr \, d\theta$$

$$= \frac{\pi}{12}$$

4. (2.5 points) Find the volume of the space region below the sphere $x^2 + y^2 + z^2 = 2$ and above the cone $z = \sqrt{x^2 + y^2}$.

$z = \sqrt{2 - x^2 - y^2}$





$x^2 + y^2 + x^2 + y^2 = 2$

$x^2 + y^2 = 1$

$$\text{Volume} = \iint_R \left(\sqrt{2 - x^2 - y^2} - \sqrt{x^2 + y^2}\right) dA$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_0^1 \left(\sqrt{2 - r^2} - \sqrt{r^2}\right) r \, dr \, d\theta$$

$$= 2\pi \left(\frac{2^{3/2}}{3} - \frac{2}{3}\right)$$

≈ 1.735

Quiz 8 - Problem 1

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(%i20) 'integrate('integrate(2*x-y,x,y^2,2+y),y,-1,2);
```

$$(\%o20)\quad \int_{-1}^{2}\int_{y^2}^{y+2} 2\,x-y\,dx\,dy$$

```
(%i21) ev(%,nouns);
```

$$(\%o21)\quad \frac{243}{20}$$

Quiz 8 - Problem 2

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(%i22) 'integrate('integrate(x^2+y,y,x^2,2-x^2),x,-1,1);
```

$$(\%o22)\quad \int_{-1}^{1}\int_{x^2}^{2-x^2} y+x^2\,dy\,dx$$

```
(%i23) ev(%,nouns);
```

$$(\%o23)\quad \frac{16}{5}$$

Quiz 8 - Problem 3

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(%i24) 'integrate('integrate(r,r,0,sin(3*theta)),theta,0,%pi/3);
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$$(\%o24)\quad \int_{0}^{\frac{\pi}{3}}\int_{0}^{\sin(3\,\theta)} r\,dr\,d\theta$$

```
(%i25) ev(%,nouns);
```

$$(\%o25)\quad \frac{\pi}{12}$$

Quiz 8 - Problem 4

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(%i30) 'integrate('integrate((sqrt(2-r^2)-sqrt(r^2))*r,r,0,1),theta,0,2*%
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$$(\%o30)\quad 2\,\pi\int_{0}^{1} r\left(\sqrt{2-r^2}-\left|r\right|\right)dr$$

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(%i31) ev(%,nouns);
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$$(\%o31)\quad 2\left(\frac{2^{3/2}}{3}-\frac{2}{3}\right)\pi$$