Math 173 - Quiz 6

March 14, 2013

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Suppose w is a function of the three intermediate variables x, y, z, and x, y, z are functions of the two independent variables u and v. Write the chain rule formulas for $\partial w/\partial u$ and $\partial w/\partial v$.

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

2. (4 points) Compute the directional derivative of $f(x, y) = ye^{x^2} - 4y(x+1)^2 + \sin(y)$ at $(0, 0)$ in the direction of $\vec{v} = 5\hat{\imath} - 12\hat{\jmath}$.

$$\vec{\nabla} f(x,y) = \left[2xye^{x^2} - 8y(x+1)\right]\hat{\imath} + \left[e^{x^2} - 4(x+1)^2 + \cos y\right]\hat{\jmath}$$

$$\vec{\nabla} f(0,0) = 0\hat{\imath} - 2\hat{\jmath}$$

$$\|\vec{v}\| = \sqrt{5^2 + (-12)^2} = 13 \qquad \frac{1}{\|\vec{v}\|}\vec{\nabla} f(0,0) \cdot \vec{v} = \frac{1}{13}(24) = \frac{24}{13}$$

3. (2 points) Suppose z is implicitly defined as a function of x and y by the equation

$$\ln(x^2y^2 + 1) + x\cos(yz) = xz^3 + 7.$$

$F(x,y,z) = \ln(x^2y^2+1) + x\cos(yz) - xz^3$

Find $\dfrac{\partial z}{\partial y}$.

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z} = \frac{-\left(\frac{2x^2y}{x^2y^2+1} - xz\sin(yz)\right)}{-xy\sin(yz) - 3xz^2}$$

4. (2 points) Let $f(x, y) = 6 - 4x^2 - 8y^2$. Find a vector in the direction of maximum increase of f at $(2, 1)$.

$$\vec{\nabla} f(x,y) = -8x\hat{\imath} - 16y\hat{\jmath}$$

$$\vec{\nabla} f(2,1) = -16\hat{\imath} - 16\hat{\jmath}$$