Math 173 - 1st Final Exam

Name _____

May 3, 2012

Score _____

Show all work. Supply explanations when necessary. **Unless otherwise specified, you may use your calculator to evaluate any integrals.** Each problem is worth 10 points.

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1. Find $\vec{r}(t)$ given that $\dfrac{d\vec{r}}{dt} = \dfrac{1}{1+t^2}\hat{\imath} + \dfrac{1}{t^2}\hat{\jmath} + \dfrac{1}{t}\hat{k}$ and $\vec{r}(1) = 2\hat{\imath}$.

2. Find a vector of magnitude 5 that is orthogonal to both $\vec{x} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{y} = -\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$.

3. Find the limit or show that it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y}{x+y}$

(b) $\lim_{(x,y)\to(2,2)} \dfrac{x-y}{x^2-y^2}$

4. Find a set of parametric equations for the line tangent to the graph of $\vec{r}(t)$ at the point $(e, 0, 2)$.

$$\vec{r}(t) = te^t\hat{\imath} + \sin(\pi t)\hat{\jmath} + \sqrt{3+t^2}\hat{k}$$

5. Find the directional derivative of $g(x, y, z) = xye^z$ at $(2, 4, 0)$ in the direction of $(0, 0, 0)$.

Follow-up: At the point $(2, 4, 0)$, in what direction is g increasing most rapidly?

6. Use the chain rule to find $\dfrac{\partial w}{\partial s}$ when $s = 4$ and $t = \pi/4$.

$$w = 5x^3 - xy^2; \qquad x = s\cos t, \quad y = s\sin t$$

7. Sketch the region R whose area is given by the iterated integral. Then reverse the order of integration and evaluate the new iterated integral by hand.

$$\int_0^4 \int_0^{x/2} dy\, dx \; + \; \int_4^6 \int_0^{6-x} dy\, dx$$

8. Find and classify all critical points of the function $f(x, y)$.

$$f(x, y) = -x^2 - 5y^2 + 10x - 10y - 28$$

9. Find the angle between the planes (i.e. the angle between the normal vectors).

$$x - 3y + 6z = 4$$
$$5x + y - z = 4$$

10. Find an equation of the plane tangent to the graph of

$$f(x, y) = (x + 1)^2 e^{2y} + y \sin(x + \pi/2)$$

at the point where $(x, y) = (0, 0)$.

11. Let E be the bounded region between the graphs of $y = 4x - x^2$ and $y = x^2$. Find the average value of $f(x, y) = x + y$ over E.

12. Find the unit tangent vector at the point where $t = \pi/2$.

$$\vec{r}(t) = 2\sin t\hat{\imath} + 2\cos t\hat{\jmath} + 4\sin^2 t\hat{k}$$

13. Consider the surface described by the equation $z = x^2 + y^2 + 3$.

(a) Sketch or describe (in detail) the level curve $z = 4$.

(b) Sketch or describe (in detail) the level curve $y = 2$.

(c) Sketch the surface.

14. Let $\vec{u}$ be the vector from $(6, 3, 1)$ to $(8, 0, 4)$. Let $\vec{v}$ be the vector in the xy-plane with magnitude 4 that makes an angle of 30° with the positive x-axis. Find $\text{proj}_{\vec{u}}\,\vec{v}$.

15. The solid region inside the cylinder $x^2 + y^2 = 2$ is bounded below by the surface $z = 0$ and above by $z = x^2 + y^2 + 3$. The density of the solid at the point (x, y, z) is given by $\rho(x, y, z) = y + z^2 + 1$. Find the mass of the solid. Use your calculator to evaluate the integral.