Math 173 - Quiz 5

February 23, 2012

Name key

Score

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Use the formula involving $\vec{r}'$ and $\vec{r}''$ to compute the curvature function for $f(x) = x^3 - 10x + 1$. Verify that you get the same result by using the formula involving $f'(x)$ and $f''(x)$.

$$x = t, \quad y = t^3 - 10t + 1 \Rightarrow \vec{r}(t) = t\,\hat{i} + (t^3 - 10t + 1)\,\hat{j}$$

$$\vec{r}\,'(t) = \hat{i} + (3t^2 - 10)\,\hat{j}$$

$$\vec{r}\,''(t) = 6t\,\hat{j}$$

$$|\vec{r}\,'(t)| = \sqrt{1 + (3t^2 - 10)^2}$$

$$\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3t^2-10 & 0 \\ 0 & 6t & 0 \end{vmatrix} = 6t\,\hat{k} \Rightarrow |\vec{r}\,' \times \vec{r}\,''| = 6|t|$$

$$\boxed{K = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3} = \frac{6|t|}{\left(\sqrt{1 + (3t^2 - 10)^2}\right)^3}}$$

SAME THING!

Using OTHER FORMULA ...

$$f(x) = x^3 - 10x + 1$$

$$f'(x) = 3x^2 - 10$$

$$f''(x) = 6x$$

$$\boxed{K = \frac{6|x|}{\left[1 + (3x^2 - 10)^2\right]^{3/2}}}$$

2. (8 points) Page 878, Problem #20

$$\vec{r}(t) = (4\sin t - 4t\cos t)\hat{\imath} + (4\cos t + 4t\sin t)\hat{\jmath} + \frac{3}{2}t^2\hat{k}$$

$$\vec{r}\,'(t) = 4t\sin t\,\hat{\imath} + 4t\cos t\,\hat{\jmath} + 3t\,\hat{k}$$

$$|\vec{r}\,'(t)| = \sqrt{16t^2\sin^2 t + 16t^2\cos^2 t + 9t^2} = \sqrt{25t^2} = 5t, \text{ ASSUMING } t > 0$$

a) $$s = \int_0^t 5\tau\, d\tau = \frac{5}{2}\tau^2 \Big|_0^t = \frac{5}{2}t^2$$

b) $$s = \frac{5}{2}t^2 \Rightarrow t = \sqrt{\frac{2}{5}s}$$

$$\vec{r}(s) = \left(4\sin\sqrt{\frac{2s}{5}} - 4\sqrt{\frac{2s}{5}}\cos\sqrt{\frac{2s}{5}}\right)\hat{\imath} + \left(4\cos\sqrt{\frac{2s}{5}} + 4\sqrt{\frac{2s}{5}}\sin\sqrt{\frac{2s}{5}}\right)\hat{\jmath} + \frac{3}{5}s\,\hat{k}$$

c) $$\vec{r}(s)\Big|_{s=\sqrt{5}} \approx 1.03\hat{\imath} + 5.41\hat{\jmath} + 1.34\hat{k} \Rightarrow (1.03, 5.41, 1.34)$$

$$\vec{r}(s)\Big|_{s=4} \approx 2.29\hat{\imath} + 6.03\hat{\jmath} + 2.4\hat{k} \Longrightarrow (2.29, 6.03, 2.4)$$

d) $$\vec{r}\,'(s) = 4\sqrt{\frac{2s}{5}}\sin\left(\sqrt{\frac{2s}{5}}\right)\left(\frac{1}{2}\right)\left(\frac{2s}{5}\right)^{-1/2}\left(\frac{2}{5}\right)\hat{\imath} + 4\sqrt{\frac{2s}{5}}\cos\sqrt{\frac{2s}{5}}\left(\frac{1}{2}\right)\left(\frac{2s}{5}\right)^{-1/2}\left(\frac{2}{5}\right)\hat{\jmath} + \frac{3}{5}\hat{k}$$

$$= \frac{4}{5}\sin\sqrt{\frac{2s}{5}}\,\hat{\imath} + \frac{4}{5}\cos\sqrt{\frac{2s}{5}}\,\hat{\jmath} + \frac{3}{5}\hat{k}$$

$$|\vec{r}\,'(s)| = \sqrt{\frac{16}{25} + \frac{9}{25}} = 1$$