Math 173 - 1st Final Exam

May 5, 2011

Name ____________________
Score ________________

Show all work. Supply explanations when necessary. **Unless otherwise specified, you may use your calculator to evaluate any integrals.** Each problem is worth 10 points.

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1. Sketch the region of integration, reverse the order of integration, and **evaluate the iterated integral by hand**.

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{x}{y^5+1}\, dy\, dx$$

2. Find and classify all relative extreme values of the function $f(x, y)$.

$$f(x, y) = x^2 + xy + 2y^2 + x - 3y + 10$$

3. A plane passes through the points $P\,(2, 1, 3)$, $Q\,(-7, 6, -1)$ and $R\,(3, 0, -1)$. Find a set of parametric equations for the line normal to the plane and passing through $(2, 1, 3)$.

4. Find a vector of length 3 that is normal to the plane given by $5x - 3y + 2z = 10$.

5. Let $\vec{r}(t) = t^2\hat{\imath} + (\sin t - t\cos t)\hat{\jmath} + (\cos t + t\sin t)\hat{k}$. Find the unit tangent vector and the principal unit normal vector. (Hint: If you're doing everything correctly, the computations should not be messy.)

6. Let $\vec{a} = 5\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} - 4\hat{\jmath} + 2\hat{k}$. Now let $\vec{g} = \text{proj}_{\vec{c}}\,\vec{a}$. Compute $\vec{g}$ and then compute $\vec{g} + \vec{c}$.

7. Let E be the space region bounded by the paraboloids $z = 2x^2+2y^2$ and $z = 6-x^2-y^2$. Evaluate the following triple integral. (Hint: It's easier in cylidrical coordinates.)

$$\iiint\limits_E (x^2 + y^2)\, dV$$

8. A batter hits a baseball 3 ft above the ground toward the center field fence, which is 10 ft high and 400 ft from home plate. The ball leaves the bat with a speed of 115 ft/sec at an angle of 50° above the horizontal. Does the ball clear the fence? (Use $g = 32\,\text{ft/sec}^2$.)

9. Use Lagrange multipliers to find the extreme values of $f(x, y) = \frac{1}{3}x^3 + y^2$ on the unit circle $x^2 + y^2 = 1$.

10. The curves described by the vector-valued functions $\vec{r_1}(t) = t\hat{\imath} + (t^2 + 3)\hat{\jmath} + (6 - 5t)\hat{k}$ and $\vec{r_2}(t) = t^2\hat{\imath} + (2t + 2)\hat{\jmath} + t^3\hat{k}$ intersect at the point $(1, 4, 1)$. Find the angle of intersection of the curves. (Hint: Find the angle between the velocity vectors at the point of intersection.)

11. The temperature at a point (x, y) is given by

$$T(x, y) = \frac{xy}{x^2 + y^2 + 1}$$

where T is measured in °C and x and y in meters.

(a) At the point $(1, 1)$, in what direction does the temperature **decrease** the fastest?

(b) Find the rate of change of temperature at the point $(1, 1)$ in the direction toward the point $(2, 3)$.

12. Show that the limit does not exist. (Hint: $y = x^2$ is a path through $(0, 0)$.)

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$$

13. Set up the definite integral that gives the length of the space curve defined by the following vector-valued function over the interval from $t = 0$ to $t = 5$. Use your calculator to approximate the value of the integral.

$$\vec{r}(t) = \sin(3t)\hat{\imath} + t\cos(t)\hat{\jmath} + e^{-5t}\hat{k}$$

14. Suppose u is a function of x, y, and z, where x, y, and z are functions of r, s, and t. State the chain rule formulas for $\partial u/\partial r$ and $\partial u/\partial t$.

15. A thin plate is bounded by the graphs of $y = x^2$ and $y = 2 - x$. The density of the plate at the point (x, y) is given by $\rho(x, y) = 5 + xy$. Find the center of mass of the plate. (Be sure to type $x * y$ on your calculator.)