Math 173 - Quiz 5

Name key

Score ____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Find a set of parametric equations for the line tangent to the graph of $\vec{r}(t)$ at the given point.

$$\vec{r}(t) = t\hat{i} + t^2\hat{j} - 3t\hat{k}, \quad P(0,0,0)$$

$$\vec{\Gamma}'(t) = \hat{i} + \partial t\hat{j} - 3\hat{k}$$

$$(0,0,0) \quad \text{corresponds to } t = 0$$

$$\vec{\Gamma}'(0) = \hat{i} - 3\hat{k} = \text{Direction Vectors}$$

$$(0,0,0) = \text{Point}$$

$$X = t$$

$$y = 0$$

$$z = -3t$$

2. (5 points) To open the 1992 Summer Olympics in Barcelona, bronze medalist archer Antonio Rebollo lit the Olympic torch with a flaming arrow. Suppose that Rebollo shot the arrow at a height of 6 ft above the ground 90 ft from a 70-ft-high cauldron, and he wanted the arrow to reach its maximum height exactly 4 ft above the center of the cauldron. Find the initial speed and firing angle of the arrow.

$$\vec{r}(t) = V_0 \cos \theta t \hat{c} + (-16t^2 + V_0 \sin \theta t + 6)\hat{j}$$

SIMULTANEOUSLY, WE MUST HAVE

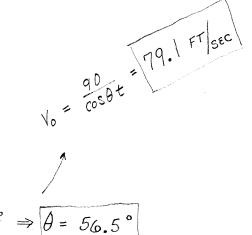
$$V_{0}\cos\theta t = 90$$

$$-16t^{2} + V_{0}\sin\theta t + 6 = 74$$

$$-32t + V_{0}\sin\theta t = 0$$

$$t = 2.06155$$

$$\frac{V_{0}\sin\theta t = 68 + 16t^{2}}{V_{0}\cos\theta t = 90} \Rightarrow Tan\theta = \frac{136}{90} \Rightarrow \theta = 56.5^{\circ}$$



3. (3 points) Find the unit tangent vector for following vector-valued function.

$$\vec{r}(t) = (e^{t}\cos t)\hat{i} + (e^{t}\sin t)\hat{j} + 2\hat{k}$$

$$\vec{r}'(t) = (e^{t}\cos t - e^{t}\sin t)\hat{i} + (e^{t}\sin t + e^{t}\cos t)\hat{j}$$

$$|\vec{r}'(t)| = \left[e^{at}\cos^{2}t - ae^{at}\sin t\cos t + e^{at}\cos^{2}t + e^{at}\sin^{2}t + ae^{at}\sin t\cos t + e^{at}\cos^{2}t\right]^{1/2}$$

$$= \sqrt{a}e^{at} = \sqrt{a}e^{t}$$

$$\vec{r}'(t) = \frac{1}{\sqrt{a}}\left((\cos t - \sin t)\hat{i} + (\sin t + \cos t)\hat{j}\right)$$