Math 172 - Quiz 6

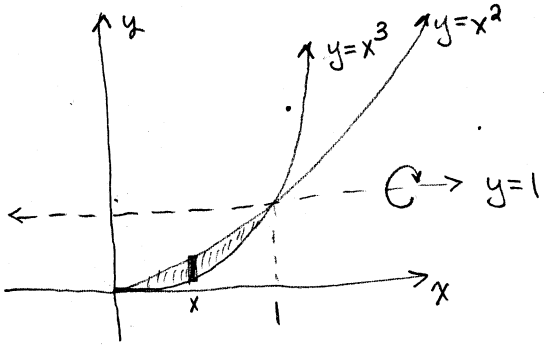
October 4, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) The 1st-quadrant region bounded by the graphs of $y = x^2$ and $y = x^3$ is rotated about the line $y = 1$ to form a solid. Find its volume.



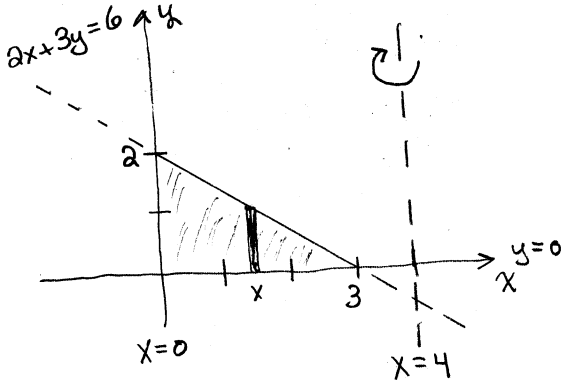
WASHERS...

$$\pi \int_0^1 (1-x^3)^2 - (1-x^2)^2 \, dx$$

$$= \pi \int_0^1 \left(-2x^3 + x^6 + 2x^2 - x^4 \right) dx$$

$$= \pi \left(-\frac{2}{4} + \frac{1}{7} + \frac{2}{3} - \frac{1}{5} \right) = \boxed{\frac{23\pi}{210}}$$

2. (4 points) The region bounded by the graphs of $2x + 3y = 6$, $y = 0$, and $x = 0$ is rotated about the line $x = 4$ to form a solid. Find the volume of the solid.



SHELLS...

$$2\pi \int_0^3 (4-x)\left(\frac{6-2x}{3}\right) dx = 2\pi \int_0^3 \left(8 - \frac{14}{3}x + \frac{2}{3}x^2\right) dx$$

$$= 2\pi \left(8x - \frac{7}{3}x^2 + \frac{2}{9}x^3\right)\Big|_0^3 = 2\pi \left(24 - 21 + 6\right)$$

$$= \boxed{18\pi}$$

3. (2 points) Set up the definite integral required to find the length of the graph of $y = e^{2x}$ from the point where $x = 0$ to the point where $x = 1$. Use your calculator to approximate the value of your integral.

$$\text{ARC LENGTH} = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$= \int_0^1 \sqrt{1 + (2e^{2x})^2} \, dx$$

$$= \int_0^1 \sqrt{1 + 4e^{4x}} \, dx \approx 6.495$$