Math 172 - Quiz 5

Name key

September 27, 2017

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (2 points) Use the definitions of the hyperbolic functions (in terms of exponentials) to prove that

$$\cosh^2 x = \frac{1+\cosh 2x}{2}.$$

$$\cosh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4}$$

$$\frac{1+\cosh 2x}{2} = \frac{1 + \frac{e^{2x}+e^{-2x}}{2}}{2} = \frac{2 + e^{2x} + e^{-2x}}{4}$$

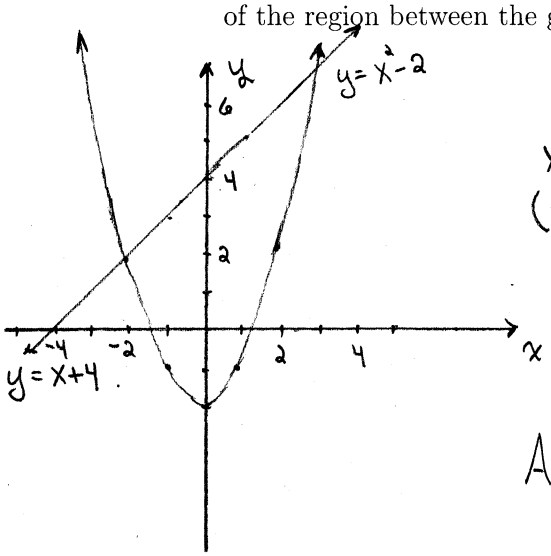
} Same.

2. (2 points) Evaluate the integral by converting to exponentials: $\int \sinh 2x\, dx$

$$\int \sinh 2x\, dx = \int \frac{e^{2x} - e^{-2x}}{2} dx = \frac{\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}}{2} + C$$

$$= \frac{1}{2}\cosh 2x + C$$

3. (6 points) Carefully sketch the graphs of $y = x^2 - 2$ and $y = x + 4$. Then find the area of the region between the graphs.



$$x^2 - 2 = x + 4$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3, \quad x = -2$$

$$\text{Area} = \int_{-2}^{3} (x+4) - (x^2-2)\, dx = \int_{-2}^{3} 6 + x - x^2\, dx$$

$$= 6x + \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_{-2}^{3} = \left(18 + \frac{9}{2} - 9\right) - \left(-12 + 2 + \frac{8}{3}\right)$$

$$= \boxed{\frac{125}{6}} = 20.8\overline{3}$$