Math 172 - Quiz 1

August 23, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary. YOU MUST WORK INDIVIDUALLY.

1. (3 points) Use implicit differentiation to determine dy/dx.

$$4xy + \ln x^2 y = 7 \Rightarrow 4xy + 2\ln x + \ln y = 7$$

$$4x\frac{dy}{dx} + 4y + \frac{2}{x} + \frac{1}{y}\frac{dy}{dx} = 0$$

$$\left(4x + \frac{1}{y}\right)\frac{dy}{dx} = -4y - \frac{2}{x} \Rightarrow \boxed{\frac{dy}{dx} = \frac{-\left(4y + \frac{2}{x}\right)}{4x + \frac{1}{y}}}$$

2. (3 points) Use logarithmic differentiation to compute $f'(2)$.

$$y = f(x) = \sqrt{\frac{x^2-1}{x^2+1}}$$

$$\ln y = \frac{1}{2}\ln(x^2-1) - \frac{1}{2}\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{x^2-1} - \frac{x}{x^2+1}$$

$$f'(x) \quad \frac{dy}{dx} = \sqrt{\frac{x^2-1}{x^2+1}}\left(\frac{x}{x^2-1} - \frac{x}{x^2+1}\right)$$

$$f'(2) = \sqrt{\frac{3}{5}}\left(\frac{2}{3} - \frac{2}{5}\right) = \frac{4}{15}\sqrt{\frac{3}{5}}$$

3. (4 points) Evaluate each integral.

(a) $\displaystyle\int \frac{(5+\ln x)^3}{2x}\,dx$

$u = 5 + \ln x$

$du = \frac{1}{x}\,dx$

$$= \frac{1}{2}\int u^3\,du = \frac{1}{8}u^4 + C$$

$$= \boxed{\frac{1}{8}(5+\ln x)^4 + C}$$

(b) $\displaystyle\int \frac{2y^2+7y-3}{y-2}\,dy$

$$= \int \left(2y + 11 + \frac{19}{y-2}\right)dy$$

$$2y + 11$$
$$y-2\,\overline{)\,2y^2+7y-3}$$
$$2y^2 - 4y$$
$$11y - 3$$
$$11y - 22$$
$$19$$

$$= \boxed{y^2 + 11y + 19\ln|y-2| + C}$$