

Math 171 - 1st Final Exam

December 2, 2010

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) Find the area of the region bounded by the graphs of $y = x^2 - 4x + 4$ and $y = 4 - x$.

2. (8 points) Use the second derivative to determine whether the graph of

$$f(x) = \cos x + \sin 9x$$

is concave up or down at the point where $x = 5$.

3. (8 points) Suppose oil spills out from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 30 m?
4. (12 points) An object is moving in such a way that its height, in feet, at time t , in seconds, is given by $s(t) = -16t^2 + 42t + 20$. Determine (a) the maximum height of the object and (b) the time when the object hits the ground.

5. (15 points) Find $\frac{dy}{dx}$.

(a) $y = 3x^2 + 7 + \cos x$

(b) $y = x^2 \sec x$

(c) $y = (1 + \sin^2 x)^2$

6. (8 points) Assume that y is implicitly defined as a function of x by the equation $\sin y + xy^2 = 1$. Find dy/dx .

7. (8 points) Evaluate the following definite integral: $\int_{-2}^2 5x^2(x^3 + 8)^2 dx$.

8. (12 points) Use the first derivative test to find open intervals on which f is increasing/decreasing.

$$f(x) = \frac{x^2 + 3x + 1}{x + 3}$$

9. (15 points) Find each limit analytically. Use ∞ , $-\infty$, or DNE if appropriate.

(a) $\lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$

(b) $\lim_{y \rightarrow 2} (2y^2 - 3y - 2)$

(c) $\lim_{\theta \rightarrow 2^-} \frac{\theta - 3}{\theta - 2}$

10. (6 points) Find the linearization of $f(x) = \sin 5x$ at $x = 0$.

(2 pts extra credit) Use your linearization to justify the following limit: $\lim_{x \rightarrow 0} \frac{\sin 5x}{2x} = \frac{5}{2}$.

11. (18 points) Consider the definite integral $\int_1^3 \frac{1}{x} dx$.

(a) In one or two complete sentences, explain why our formula for the antiderivative of x^n cannot be used to evaluate the definite integral.

(b) Use the trapezoid rule with four subintervals to approximate the value of the integral. Does your approximation over-estimate or under-estimate the exact value? How do you know?

(c) Using four subintervals of equal length and left endpoints, compute a Riemann sum corresponding to the definite integral.

12. (8 points) Evaluate the following indefinite integral: $\int \left(\sqrt[3]{y^2} + \frac{1}{y^2} + \csc^2 y \right) dy$

13. (20 points) Do any TWO of the following problems.

(a) Use the limit definition of the derivative to find $f'(x)$ if $f(x) = x - x^2$.

(b) State the Mean Value Theorem (MVT). Then apply the MVT to the function $f(x) = x^2$ on $[0, 3]$ and find the appropriate number c .

(c) Use Newton's method to find either one of the two points where the graph of $y = \cos x$ intersects the graph of $y = x^2$. Show each one of your estimates, rounded to the nearest ten-thousandth.

(d) Using the definition of continuity, describe three reasons why a function may fail to be continuous at a point.