

Math 171 - Test 2

October 21, 2010

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) If $g(x) = 7x^3(x - 4x^2)$, find the value of $g''(2)$.

2. (8 points) Let $f(x) = (\sin x)^4$. Find $f'(x)$. Then find an equation of the line tangent to the graph of f at the point where $x = \pi/4$.

3. (12 points) An object is launched from a tall building so that its height in feet after t seconds (measured from the ground) is given by $s(t) = -16t^2 + 64t + 512$.

(a) What is the velocity of the object after 7 seconds?

(b) What is the maximum height of the object?

(c) When does the object hit the ground?

4. (8 points) Assume that y is implicitly defined as a function of x by the following equation:

$$xy = 2x - y^3.$$

Find $\frac{dy}{dx}$ at $(1, 1)$.

5. (5 points) Suppose that the infected region of an injury is circular, and its radius is growing at a rate of 1.3 mm/hr. Find the rate of change of area of the infected region when the radius is 3 mm.
6. (8 points) Find the absolute maximum and minimum values of $g(x) = x^3 - 3x^2 + 1$ on the interval $[-1/2, 4]$.

7. (15 points) Differentiate. Do not simplify.

(a) $\frac{d}{dw} \frac{\sqrt{w}}{w^2 + 3w + 1}$

(b) $\frac{d}{dt} (t^7 - 7t) \tan t$

(c) $\frac{d}{dx} \cos(x^3 + x)$

8. (6 points) Find all critical numbers of the function $f(x) = 5x^{2/3} + x^{5/3}$.

9. (3 points) If $f'(5) = 7$, $f(5) = 0$, $g(1) = 5$, and $g'(1) = 3$, determine $\frac{d}{dx} f(g(x))$ when $x = 1$.

10. (15 points) Consider the function $f(x) = x^4 + 4x^3 - 36x^2$.

(a) Determine $f'(x)$.

(b) Find all x -values for which f' is zero or not defined.

(c) Determine open intervals on which f is increasing/decreasing.

(d) Identify all relative extreme values of f .

(e) Evaluate f'' at one of the critical numbers of f . How does the sign of f'' at that point support your conclusion in part (d)?

11. (6 points) What do the signs of f'' say about the graph of f ?

12. (5 points) Suppose the function f satisfies the following conditions:

- f is continuous on $[0, 5]$ and differentiable on $(0, 5)$
- $f'(x) < 2$ for all x in $[0, 5]$
- $f(0) = 0$

Use the Mean Value Theorem to show that $f(5) < 10$.

13. (3 points) What is an inflection point?