Show all work to receive full credit. Supply explanations when necessary.

- 1. (3 points) Consider the following piecewise function: $g(x) = \begin{cases} 3x^2 + ax, & x \leq 1 \\ 5 + b\cos \pi x, & x > 1 \end{cases}$
 - (a) Find a so that $\lim_{x\to 1^-} g(x) = 6$.

(b) Find b so that $\lim_{x\to 4^+} g(x) = 13$.

(c) Find all possible a and b so that $\lim_{x\to 1} g(x)$ exists.

2. (4 points) Evaluate each limit.

(a)
$$\lim_{x \to 2} \frac{x-2}{2-\sqrt{x+2}}$$

(b) $\lim_{x \to 5^+} \frac{x^2 - 2x - 15}{x^2 - 25}$

(c)
$$\lim_{x \to 0} \frac{7}{x \cot 5x}$$

3. (1 point) Suppose f is a function with the property that

$$-x^4 + 8x^3 - 24x^2 + 32x - 13 \le f(x) \le x^3 - 6x^2 + 12x - 5$$

for all x. What can be said about $\lim_{x\to 0} f(x)$?

- (a) The limit does not exist.
- (b) The limit is 3.
- (c) If it exists, the limit is between -13 and -5.
- (d) Nothing can be said about the limit.
- 4. (2 points) Sketch the graph (make it a nice graph!) of a function f such that
 - $\bullet \lim_{x \to 2^+} f(x) = 3$
 - $\lim_{x\to 2} f(x)$ exists
 - $\lim_{x \to 4} f(x)$ DNE
 - $\bullet \lim_{x \to 4^-} f(x) = -3$
 - f(2) = 0
 - f(4) = 1