

Math 157 - Final Exam

December 14, 2016

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) Use algebra to find the limit analytically.

$$\lim_{r \rightarrow -1} \frac{3(r+2)^2 + 6r + 3}{r+1}$$

2. (10 points) Let $f(x) = 1 + \sqrt[3]{2x+4}$. Compute $f'(x)$. Then use it to find an equation of the line tangent to the graph of f at the point where $x = 2$.

3. (6 points) Let $f(x) = 2/x^2$. Use at least four small intervals to estimate $f'(2)$.

4. (8 points) A bacteria culture is growing in such a way that after t hours there are $P(t) = 145e^{0.2t}$ bacteria. Compute $P(6)$ and $P'(6)$. Using units, explain what each of these values represents.
5. (10 points) Find the global (absolute) extreme values of the function $f(x) = 2x^3 - 9x^2 - 60x + 10$ on the interval $[-4, 4]$.
6. (10 points) Use the second derivative to determine whether the graph of $y = x^2e^{-x}$ is concave up or down at the point where $x = 1$.

7. (20 points) Consider the function $f(x) = 7 + 2x^3 - x^4$.

(a) Find all critical points of f .

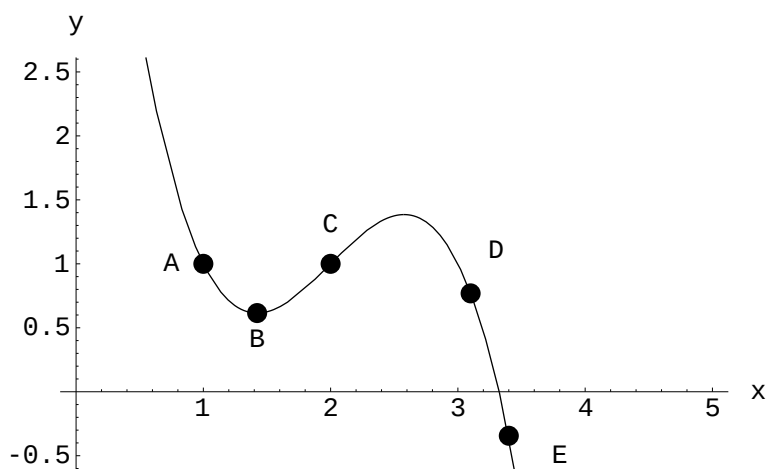
(b) Find open intervals on which f is increasing/decreasing.

(c) Find all local (relative) extreme values of f .

(d) Find open intervals on which the graph of f is concave up/down.

8. (8 points) Evaluate the indefinite integral: $\int \left(\frac{1}{x^2} - 7x^4 + e^{-2x} + 2x^{-1} \right) dx$.

9. (12 points) The graph of f is shown below. For each part of this problem, find a labeled point that satisfies the given condition.



(a) $f''(x) = 0$

(b) x is a critical point of f

(c) $f''(x) < 0$

(d) $f(x) < 0$

(e) $f'(x) > 0$

(f) $f''(x) > 0$

10. (12 points) An ice cream company finds that at a price of \$4.00, demand is 4000 units. For every \$0.25 decrease in price, demand increases by 200 units.

(a) Find the demand equation.

(b) Find a formula for the revenue.

(c) Find the price and quantity that maximize revenue. (Be sure to show how you know you've found a max.)

11. (10 points) Sketch the graph of $f(x) = 2x - 4$. Then use area (not antidifferentiation) to compute $\int_0^5 f(x) dx$.

12. (8 points) Use a left sum with 5 subintervals (rectangles) of equal width to estimate $\int_1^2 \frac{3}{x+5} dx$.

13. (8 points) Evaluate the indefinite integral: $\int 4x^3 (x^4 + 8)^5 dx$.

14. (4 points) For each part, circle the correct conclusion.

(a) If f' is increasing, then

- i. the graph of f is concave up.
- ii. the graph of f has positive slope.

(b) If $f(c) = 0$, then

- i. c is a critical point of f .
- ii. $(c, 0)$ is an x -intercept of the graph of f .

15. (6 points) The revenue from selling q items is $R(q) = 30q + 5$ and the total cost is $C(q) = 0.01q^3 - 0.7q^2 + 34q + 8$.

(a) Determine the profit function $P(q)$.

(b) Determine the marginal profit.

(c) Determine the marginal profit at $q = 25$. Based on your value should you increase or decrease production in order to increase profit? Why?

16. (10 points) Use a definite integral (and the Fundamental Theorem of Calculus) to find the area of the region under the graph of $f(x) = 8x^3 + 6x^2 + 1$ over the interval from $x = 0$ to $x = 2$.