

**Math 157 - Test 2**

October 19, 2016

Name \_\_\_\_\_

Score \_\_\_\_\_

**Show all work to receive full credit.** Supply explanations where necessary.

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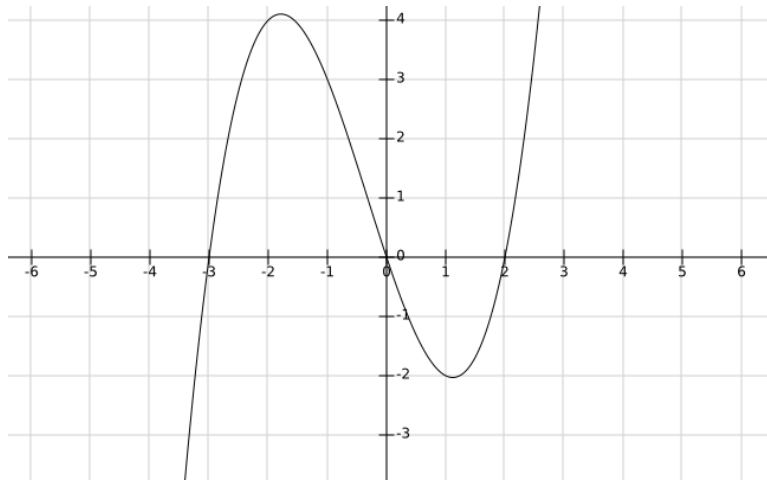
1. (8 points) The function  $p(x) = 0.07x^3 - 0.75x^2 + 1.90x + 3.10$  gives the profit in dollars from the sale of  $x$  items.

(a) Compute  $p(4)$  and  $p'(4)$ . Using units, explain what each of these values represents.

(b) Based on your results in part (a), should you sell more or fewer items to increase profit? Explain.

2. (8 points) For the function  $f(x)$ , we know that  $f(20) = 68$  and  $f'(20) = -3$ . Find an equation of the line tangent to the graph of  $f$  at the point where  $x = 20$ . Then use your tangent line equation to approximate  $f(19)$ .

3. (10 points) The graph of the function  $f(x)$  is shown below.



- (a) Use the graph to estimate  $f'(1)$ .
- (b) Find an equation of the line tangent to the graph of  $f$  at  $x = 1$ . (Because it's based on the graph, your line will only be an estimate.)
- (c) Estimate the critical points of  $f$ .
4. (8 points) Find the critical points of  $g(x) = (x^2 - 4x)^2$ .

5. (9 points) For each part of this problem, a function, its domain, and a point are given. Determine whether the point is a critical point. Explain your answer.

(a)  $f(x) = x^2$ , Domain:  $[0, 4]$ , Point:  $x = 0$

(b)  $g(x) = x^2 + 6x$ , Domain:  $[-5, 10]$ , Point:  $x = 0$

(c)  $h(x) = 2x^3 + 3x^2 - 12x + 7$ , Domain: All real numbers, Point:  $x = -2$

6. (7 points) With  $t$  in years since January 1, 2010, the population  $P$  of Slim Chance is predicted by

$$P = 35000(0.98)^t.$$

At what rate will the population be changing on January 1, 2023? Give units with your answer.

7. (20 points) Determine the derivative of each function.

(a)  $h(t) = \frac{5}{t} + \frac{8}{t^2}$

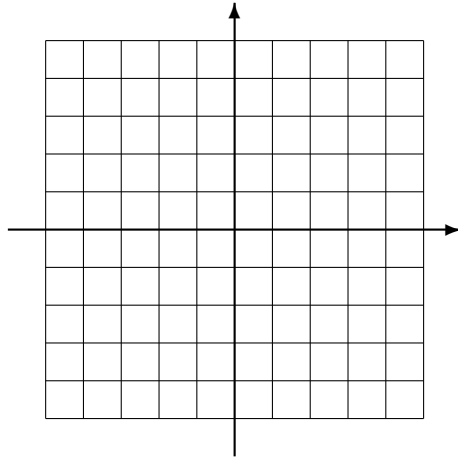
(b)  $f(x) = 1.8e^{2x} - 4.7 \ln x$

(c)  $Q = \ln(\ln t)$

(d)  $w = \frac{7y - 8y^2}{6 + 2y}$

(e)  $g(z) = \sqrt{z} e^{-z}$

8. (8 points) Sketch the graph of the function  $s(t) = 4t - t^2$ .



Use your graph to determine whether each of the following is positive, negative, or zero. Give a very brief explanation for each answer.

$s'(0)$

$s'(3)$

$s'(2)$

9. (7 points) Given that  $f(1) = 9$  and  $f'(1) = 24$ , find  $g'(1)$  if  $g(x) = \sqrt{f(x)}$ .

10. (7 points) Find the second derivative of the function  $h(x) = x^4 + e^{-3x} + \ln x$ .

11. (8 points) The table below gives the values of the functions  $f$  and  $g$  and their derivatives at selected values of  $x$ .

| $x$     | $-2$ | $-1$ | $2$  |
|---------|------|------|------|
| $f(x)$  | $1$  | $3$  | $-2$ |
| $f'(x)$ | $2$  | $-1$ | $-1$ |
| $g(x)$  | $2$  | $0$  | $-2$ |
| $g'(x)$ | $-3$ | $-2$ | $1$  |

(a) If  $h(x) = f(x) \cdot g(x)$ , use the product rule to compute  $h'(-1)$ .

(b) If  $h(x) = \frac{f(x)}{g(x)}$ , use the quotient rule to compute  $h'(2)$ .